Topological Matter

- 2D topological insulators (quantum spin-Hall effect)
- 2D topological superconductors (Majorana fermions)
- 1D topological superconductors

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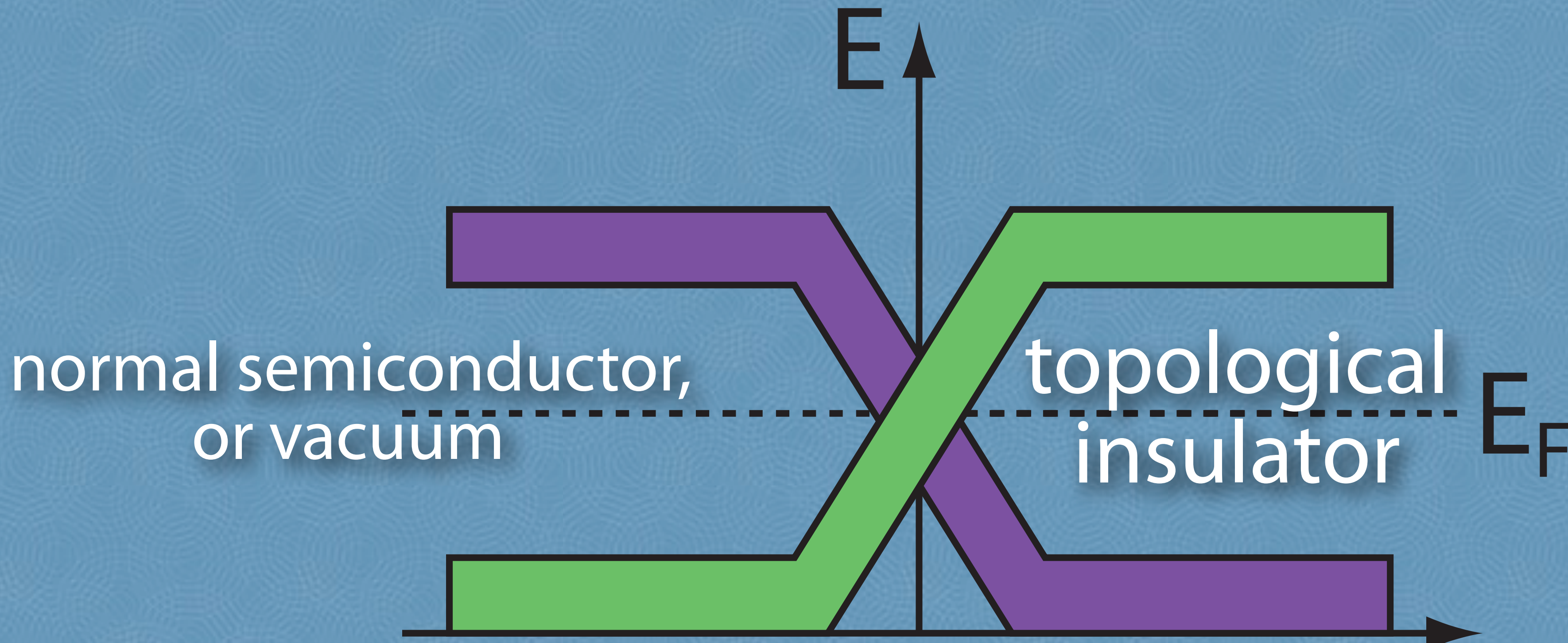
Two-dimensional massless electrons in an inverted contact

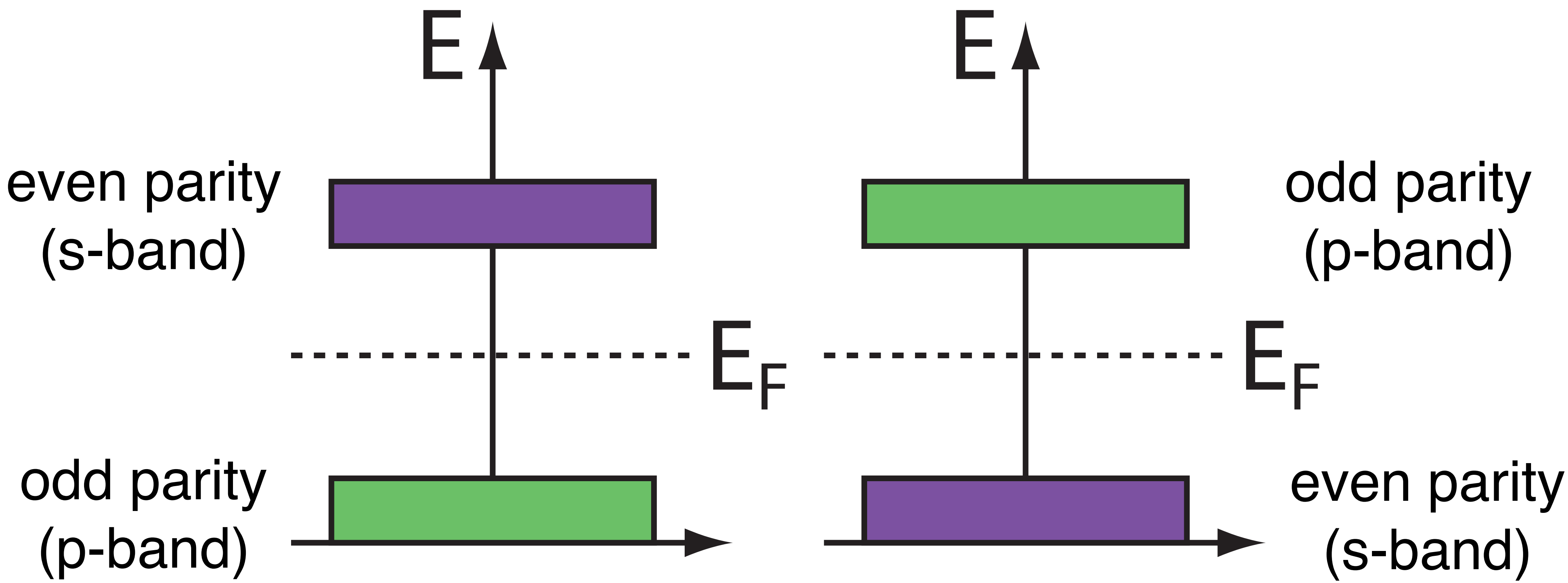
B. A. Volkov and O. A. Pankratov
P. N. Lebedev Physics Institute, Academy of Sciences of the USSR

(Submitted 20 June 1985)
Pis'ma Zh. Eksp. Teor. Fiz. **42**, No. 4, 145–148 (25 August 1985)

A new type of semiconductor structures based on the contact of two materials with mutually inverted bands is proposed. A qualitative feature of this contact is the presence of electron states which have a two-dimensional linear spectrum and which do not depend on the transition region. The properties of an inverted contact in an external magnetic field are determined.



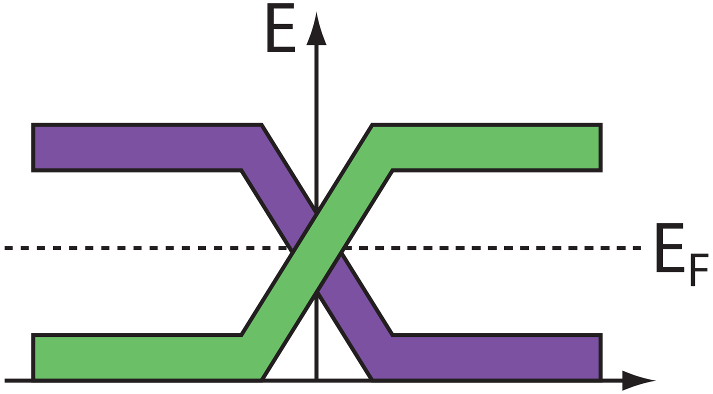
topological insulators (parity inversion)



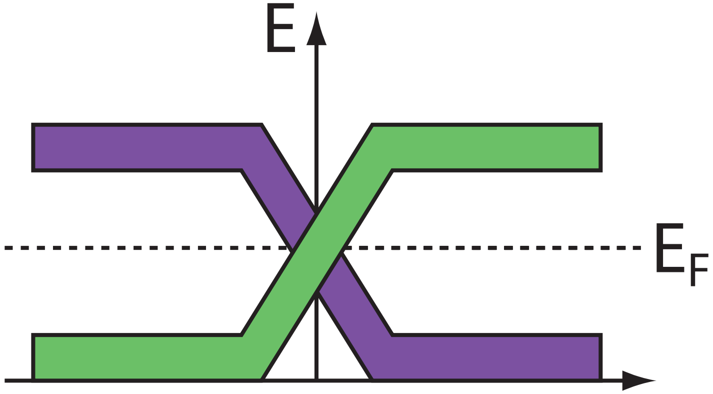
normal semiconductor
graphene :(

needed for parity inversion:
strong spin-orbit coupling (heavy atoms)

HgTe/CdTe or InAs/GaSb
quantum wells (2D),
Bi_2Se_3, Bi_2Te_3, SmB_6 (3D)
+ many, many more candidates



¿topology?

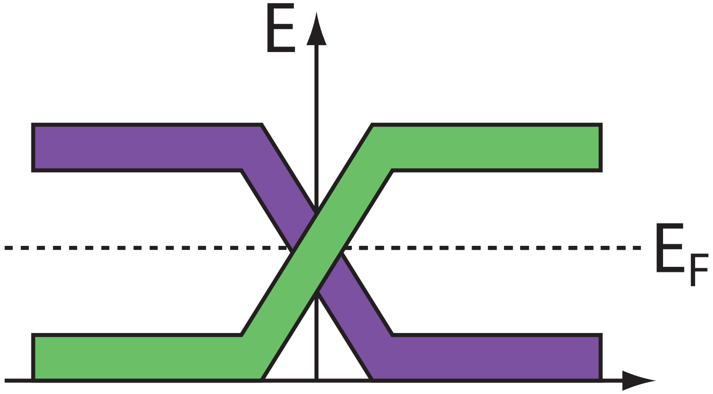


¿topology?

© Justin Wells (Aarhus)

topological insulator:
insulating bulk + gapless surface

topological consequence
of band inversion



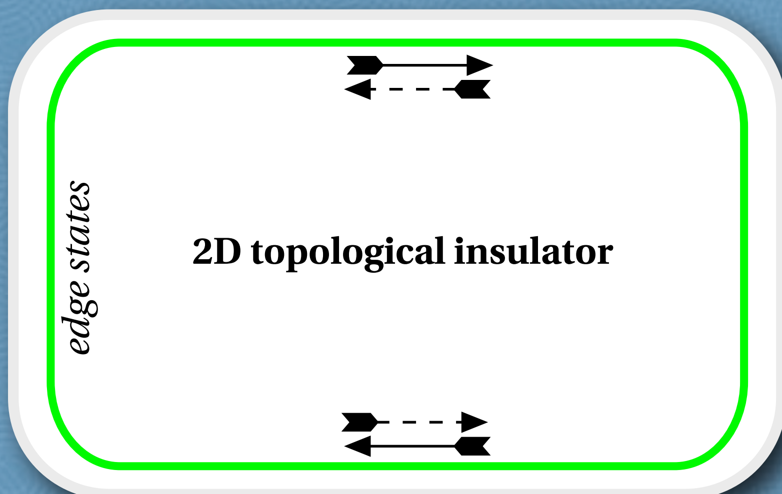
vacuum

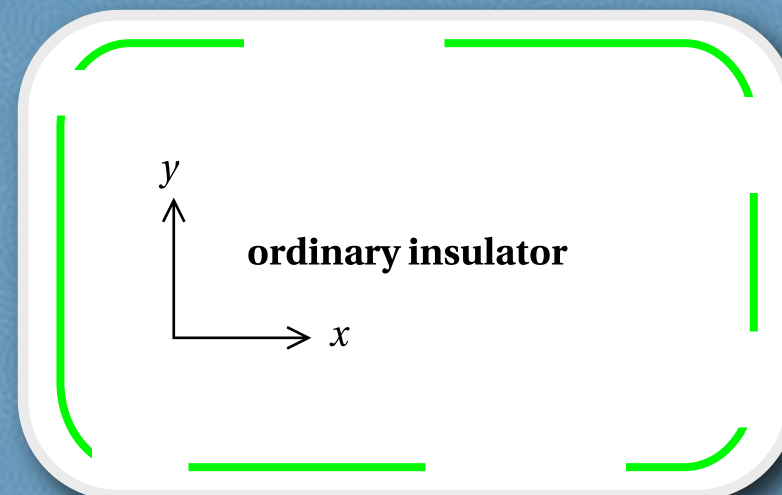
inverted band gap
semiconductor

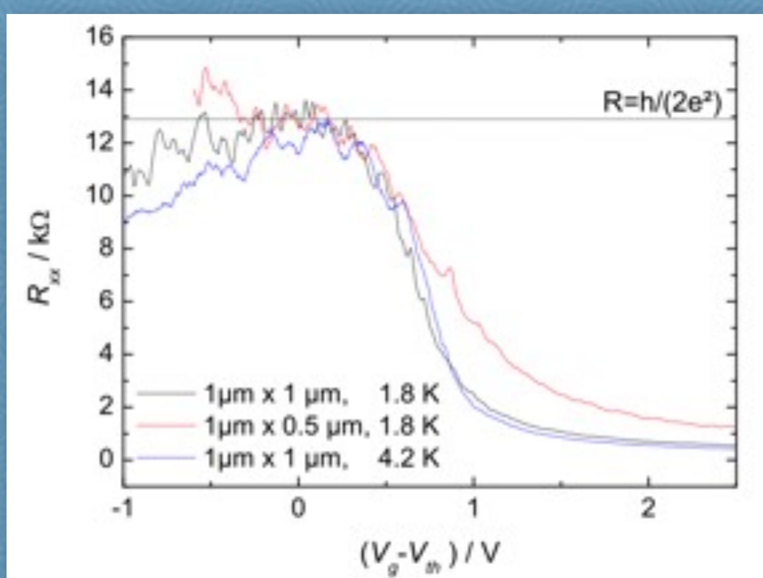
only a “bulk” inversion can remove the crossing (“topological protection”)

Stockholm, 3 September 1967

2D topological insulator
(HgTe quantum well)

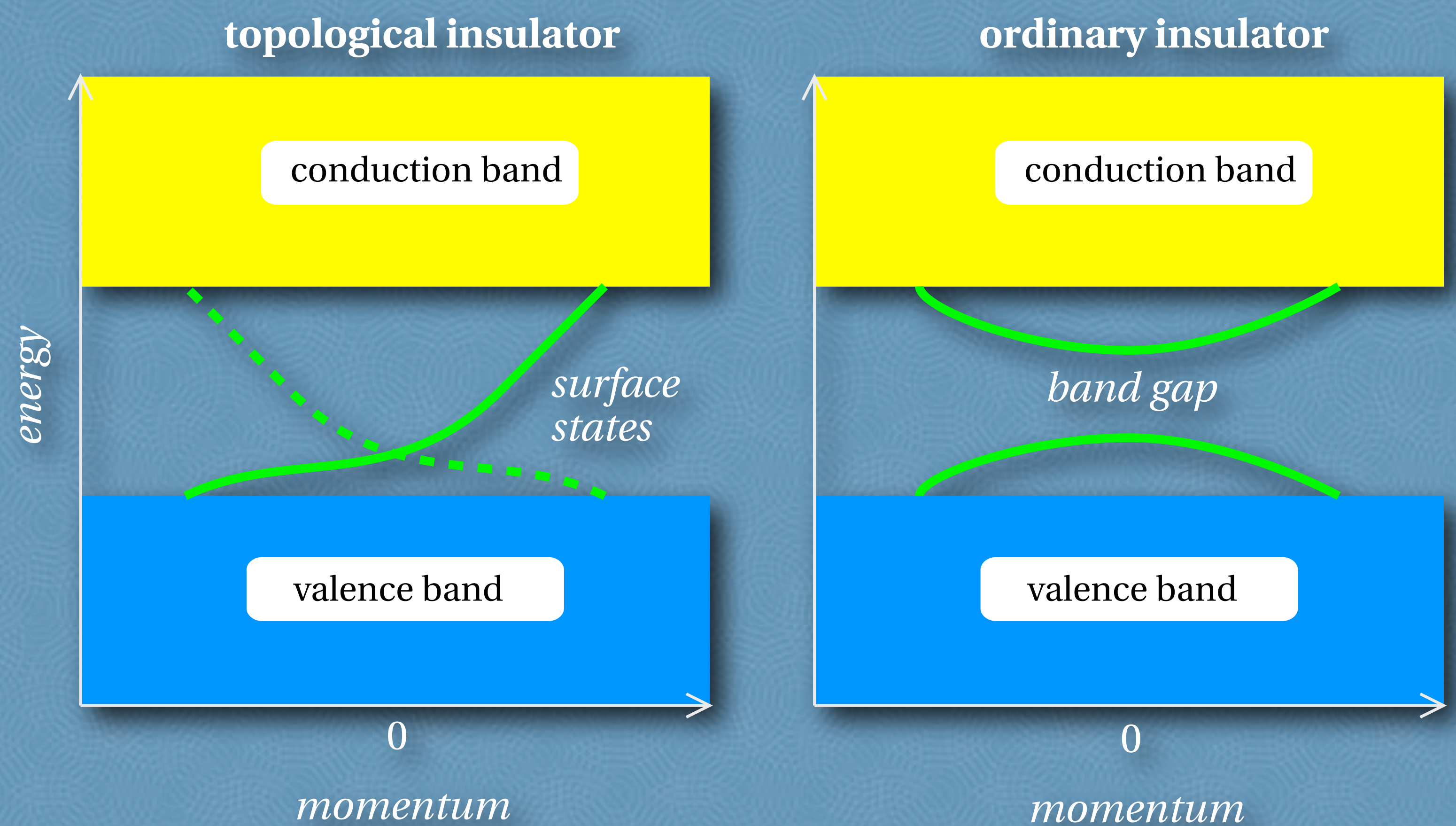






quantum spin Hall effect –
zero-field analogue of the quantum Hall effect

¿quantized conductance?



Kramers degeneracy protects the crossing at zero momentum

Fu, Kane & Mele | Moore & Balents | S.C. Zhang et al.

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Majorana fermion

From Wikipedia, the free encyclopedia

In particle physics, a **Majorana fermion** is a fermion which is its own anti-particle.

No Majorana fermions are known in nature.

Majorana fermions might *emerge* as midgap states in a superconductor

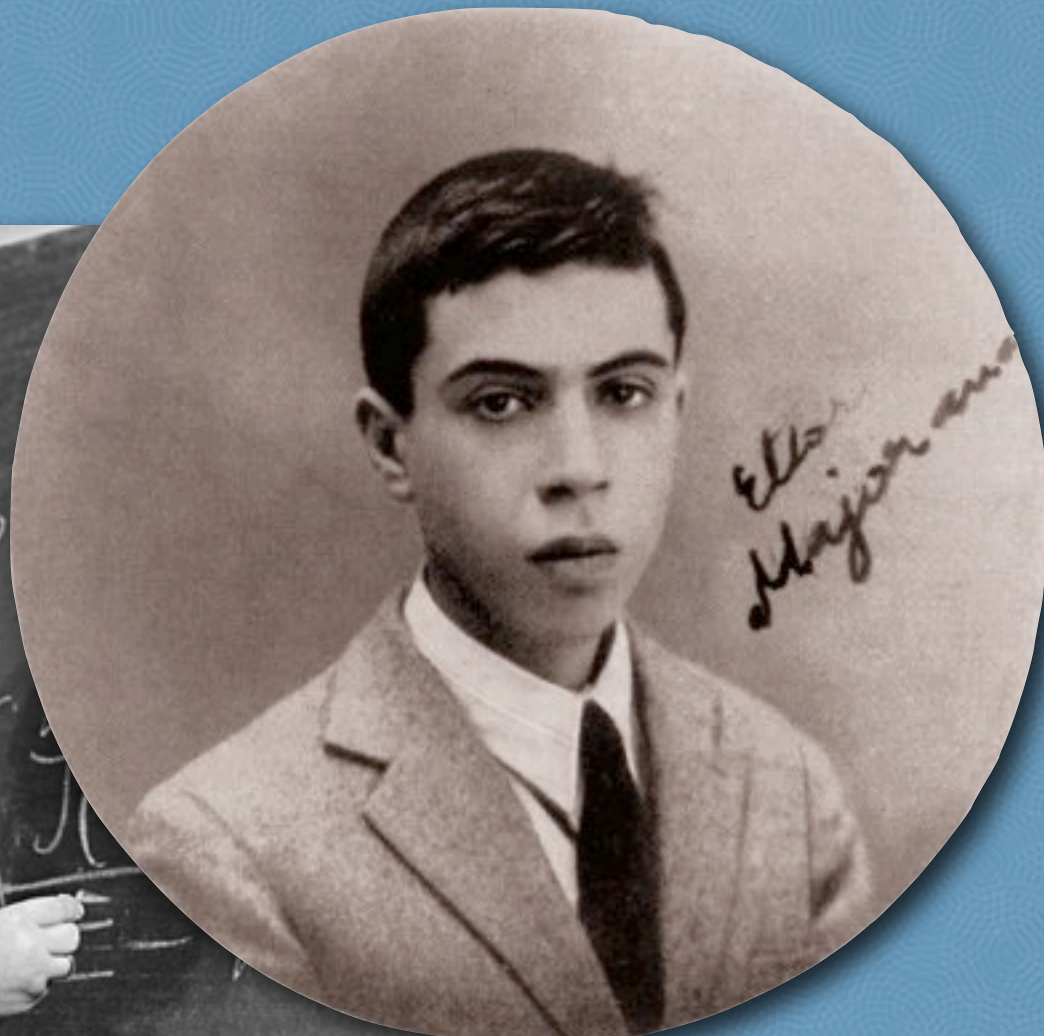
$$\gamma(-E) = \gamma^\dagger(E)$$

Volovik (1999)

Majorana fermion at E=0

(p-wave order needed to remove zero-point motion)

Bogoliubov *meets* Majorana



Bogoliubov quasiparticle

$$\gamma_{k\uparrow} = u(E)c_{k\uparrow} + u^*(-E)c^\dagger_{-k,\downarrow}$$

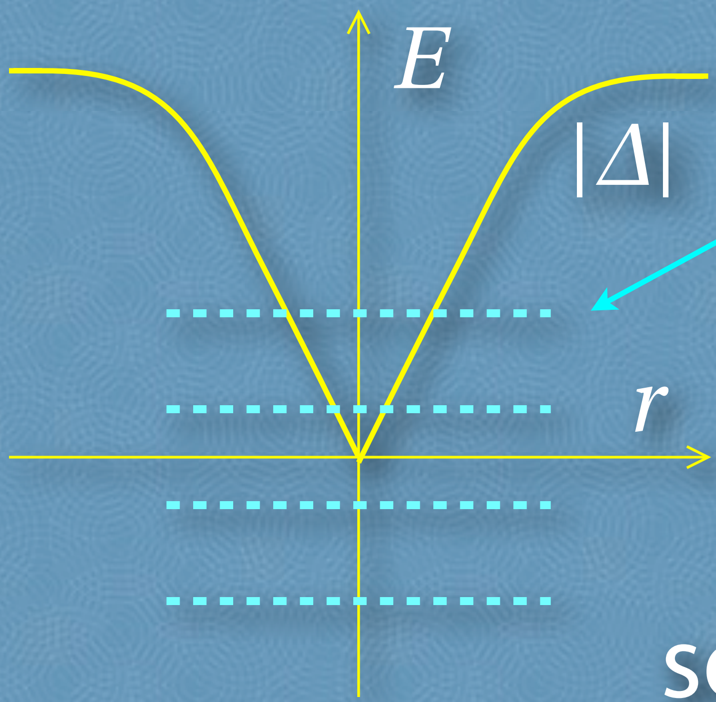
superposition of electron and hole excitations in a superconductor

spinless bound state at zero energy is a Majorana fermion

$$\gamma = \gamma^\dagger$$ particle = antiparticle

obstacle: zero-point motion

bound states in a vortex core



$$E_n = E_0(n + 1/2)$$

electron-hole symmetry:

$$\gamma(-E) = \gamma^\dagger(E)$$

so Majorana fermion at E=0

however, zero-point motion prevents a bound state at E=0

Fu & Kane (2008): use Berry phase of massless electrons to eliminate the ½ phase shift

2005: electrons in a carbon monolayer are massless (like photons) and have a “pseudospin” pointing in the direction of motion

Experimental observation of the quantum Hall effect and Berry's phase in graphene

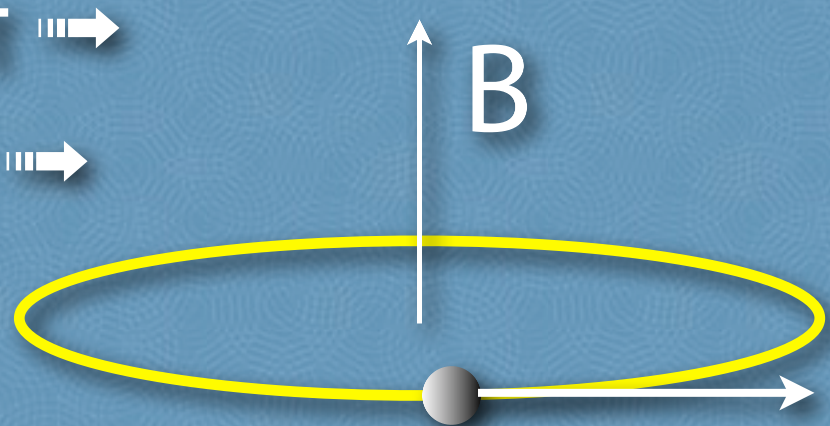
Yuanbo Zhang[1], Yan-Wen Tan[1], Horst L. Stormer[1,2] & Philip Kim[1]

K. S. Novoselov[1], A. K. Geim[1], S. V. Morozov[2], D. Jiang[1], M. I. Katsnelson[3], I. V. Grigorieva[1], S. V. Dubonos[2] & A. A. Firsov[2]

pseudospin rotates 2π

Berry phase shift of π

Landau levels shifted



the problem with graphene

- each Majorana fermion is four-fold degenerate (2x spin + 2x valley)
- nonlocality broken if spins and/or valleys can mix

for full protection from decoherence, we need massless electrons without spin or valley degeneracy

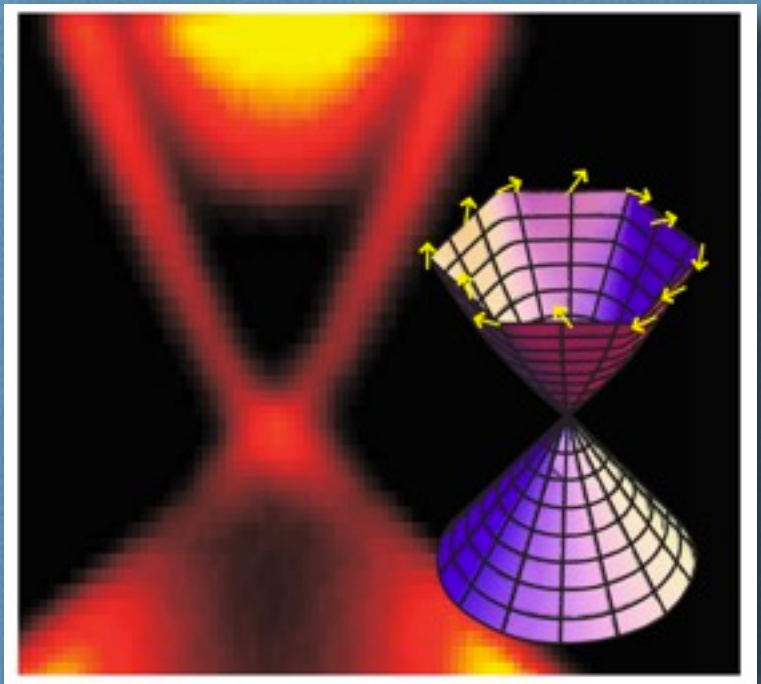
3D topological insulators to the rescue

3D topological insulator
(Bi_2Se_3, Bi_2Te_3)

insulating bulk,
metallic surface

$$H = \mathbf{p} \cdot \boldsymbol{\sigma}$$

massless Dirac
Hamiltonian
all degeneracies
removed



Hsieh, Hasan (2009)

proximity to a superconductor transforms the surface of a 3D topological insulator into a 2D topological superconductor, with Majorana fermions in vortex cores

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1D topological superconductor
(InSb nanowire)


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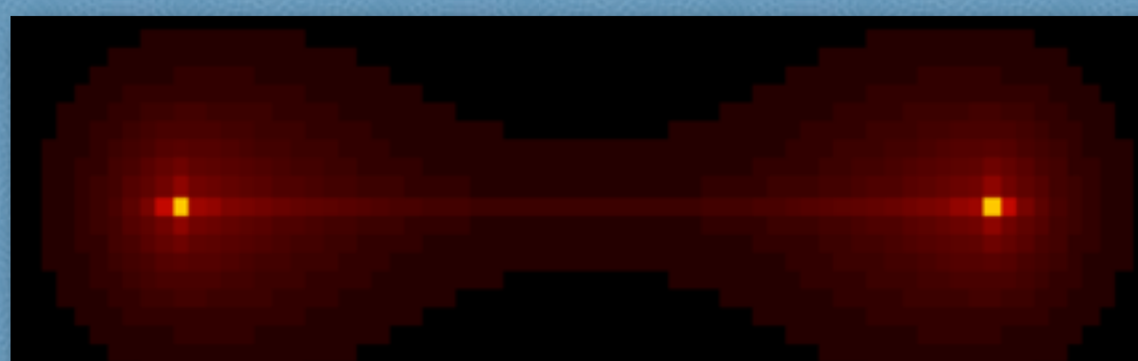
Majorana particle glimpsed in lab



AUGUST 15, 1939 PHYSICAL REVIEW VOLUME 56

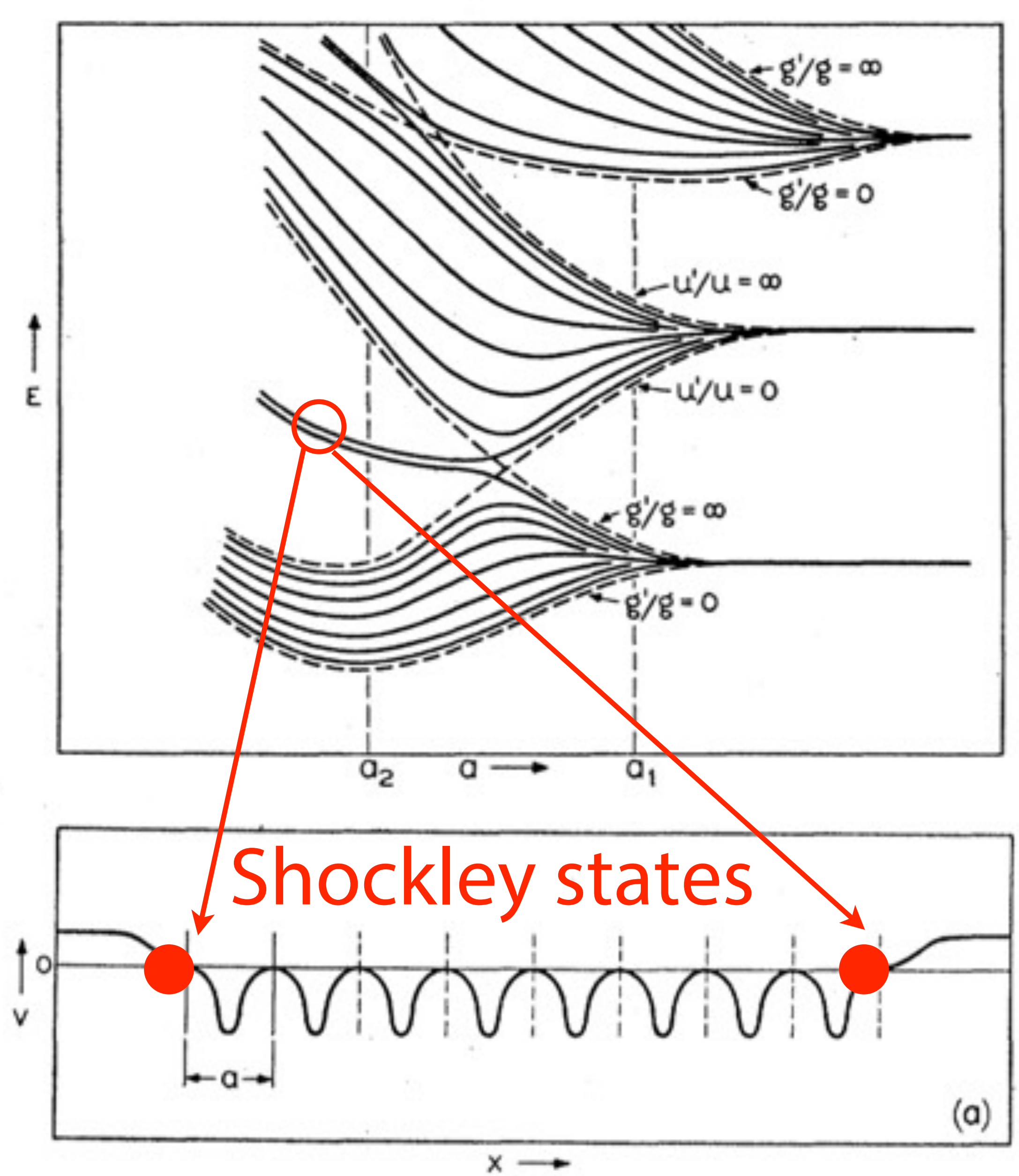
On the Surface States Associated with a Periodic Potential

WILLIAM SHOCKLEY
Bell Telephone Laboratories, New York, New York

gap closes and reopens upon formation of a line defect, leaving behind a pair of bound states at the end points

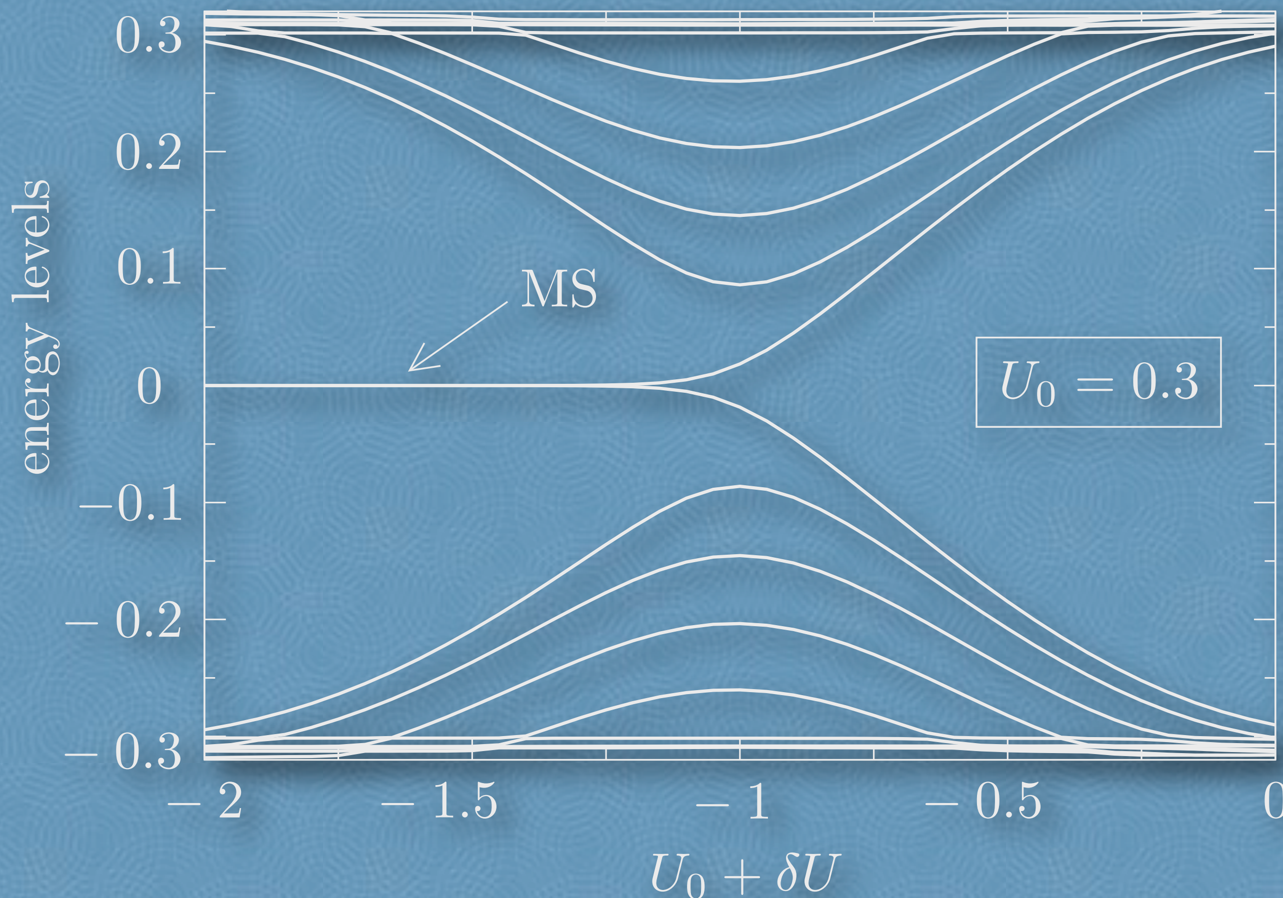
in a superconductor, electron-hole symmetry would pin these states at *E=0*

Majorana fermions!



Majorana-Shockley states in a *p*-wave superconductor

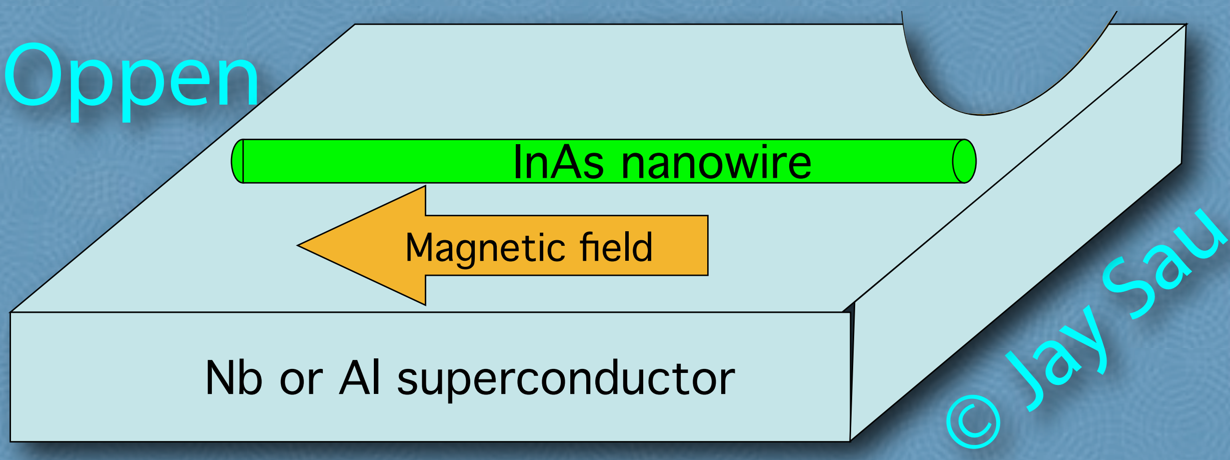
$$H = \Delta\big(p_x\sigma_x + p_y\sigma_y\big) + \big(U(\mathbf{r}) + p^2/2m\big)\sigma_z$$



p-wave from *s-wave*

Lutchyn, Sau & Das Sarma

Oreg, Refael & Von Oppen



s-wave proximity effect
Rashba spin-orbit coupling
+ Zeeman spin polarization

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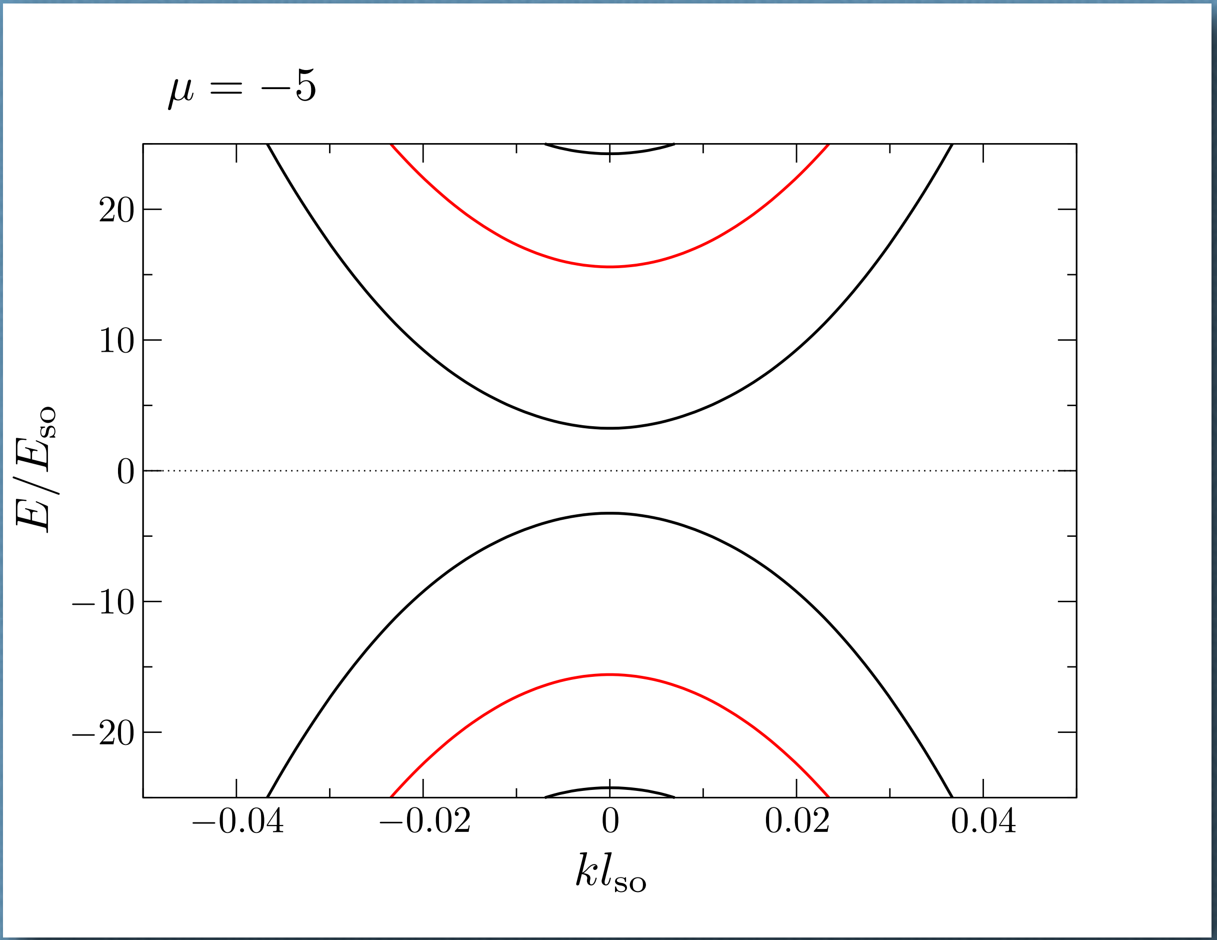
p-wave superconductor

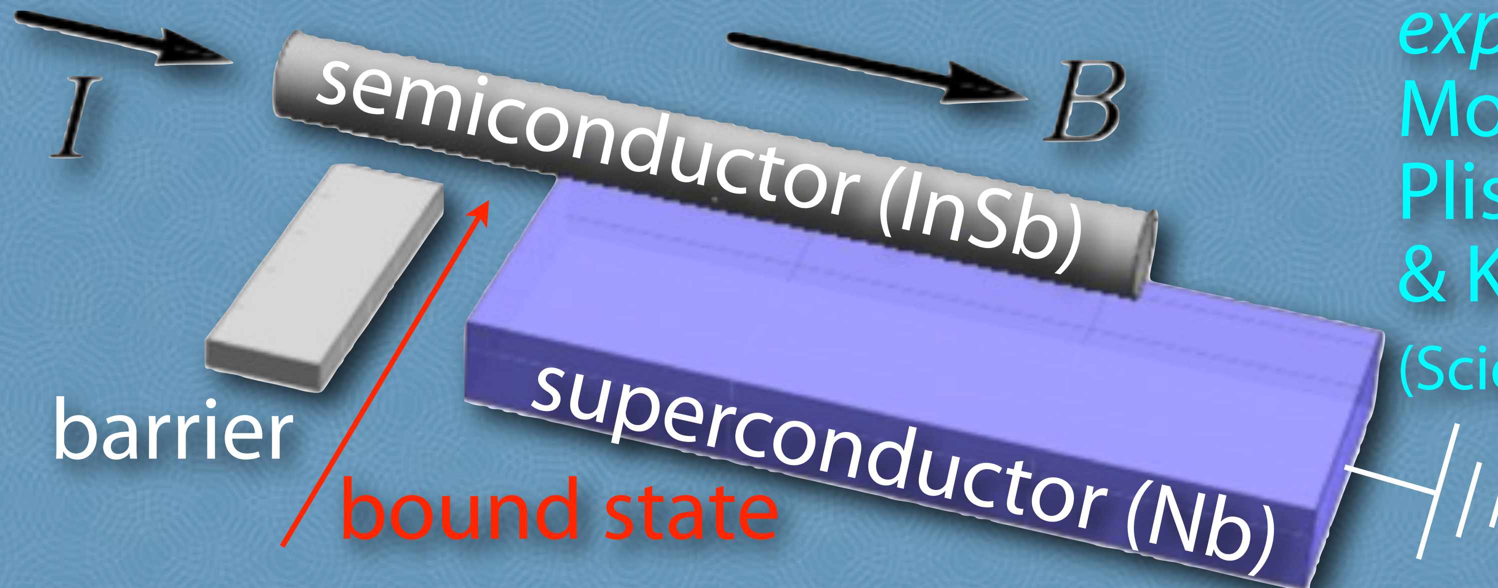


$$\mathcal{H} = \begin{pmatrix} H_{\mathrm{R}} - \mu & \Delta \\ \Delta^* & \mu - \sigma_y H_{\mathrm{R}}^* \sigma_y \end{pmatrix}$$

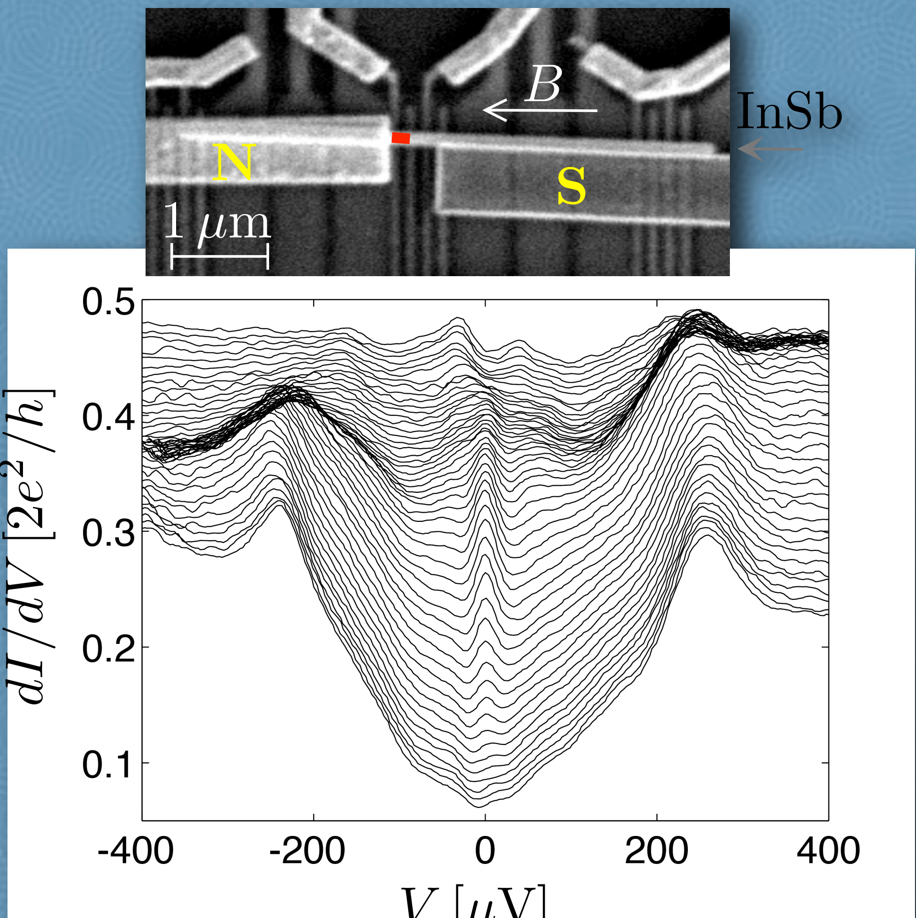
$$l_{\mathrm{so}} = \hbar^2 / m_{\mathrm{eff}} \alpha_{\mathrm{so}} \simeq 100\,\mathrm{nm}$$

$$E_{\mathrm{so}} = m_{\mathrm{eff}} \alpha_{\mathrm{so}}^2 / \hbar^2 \simeq 0.1\,\mathrm{meV}$$

$$H_{\mathrm{R}} = \frac{\boldsymbol{p}^2}{2m_{\mathrm{eff}}} + U(\boldsymbol{r}) + \frac{\alpha_{\mathrm{so}}}{\hbar}(\sigma_x p_y - \sigma_y p_x) + \tfrac{1}{2} g_{\mathrm{eff}} \mu_B B \sigma_x$$



experiment:
Mourik, Zuo, Frolov,
Plissard, Bakkers
& Kouwenhoven
(Science, April 2012)



model calculation: Wimmer & Akhmerov