Petrograd, Main Physics Observatory
Vasilyevsky island, 23 line, house 2.

15 June 1922

Dear Pavel Sigizmundovich,

Thank you very much for sending me a very interesting work of Schouten; I also will write him personally, but my proficiency with the German language leaves much more to be desired (I'm translating very slowly), and I do not have enough money to pay for translations. Due to the same reason of lack of money I do not have the ability to send you and Tatyana Alekseevna my thesis; as soon as I have the chance I will do so.

Let me express to you my thoughts about two aspects of the modern relativity principle, and I will do so through a list of separate points split into two parts.

I. On Weyl's geometry.

1. The electrodynamic equations include eight unknown variables: four *Viererpotential* components and four *Viererstrom* components. Maxwell's equations reduce to just four; the rest come out as consequences of these four. Thereby Maxwell's equations are not enough and they should be supplemented.

2. For instance, such supplementary to Maxwell's equations appears to be formed by the assumptions of Mie's theory or of Lorentz's electrons theory.

3. One of the most interesting points of view of Weyl is that he, developing Riemann's ideas, considers all material phenomena (therefore, including electrodynamics) as the world's (space-time) properties. On the basis of this point of view and considering that matter reduces to electromagnetic effects, one must admit that eight electrodynamical quantities must meet the appropriate number of geometric quantities characterizing the geometry of the world.

4. Taking Mie's position, Weyl considers only four electromagnetic variables combined into the concept of *Viererpotential* — according to this he extends the Riemannian geometry by introducing the metric connection etc.

5. If one is not taking Mie's point of view, one could try to find in the geometry two vectors, one of those would correspond to the *Viererstrom* and other to the *Viererpotential*.

6. As you probably have seen in my paper, I tried to track down such vectors by means of two ways: either by considering a flat hypersurface or by analyzing the variation of the angles of both the cogradient as well as the vectors under their parallel transport.

7. The development of these considerations had led me to the fundamental tensor g_{ik} and to two scale vectors ϕ_i and f_i — rather than the single one of Weyl; furthermore it proved to be possible to construct a number of invariants (both coordinate and scale, as well as integral) out of g_{ik}, ϕ_i, and f_i.

8. In his letter to me from 15 May 1922 Weyl remarks on the futility of attempts of unnecessary generalizations. I am holding the same opinion, but it seemed to me not useless to attempt to find the properties of the world's geometry that allow us to get rid of Mie's restrictive theory.

   These 8 items were developed in my paper that I have sent you (I do not know whether you have received it). After finishing that article I have obtained some extra results that I would like to report to you as well:

9. Inclusion of the second scale vector into the world function gives us besides the 10 equations for g_{ik} and the four equations obtained under variation of ϕ_i, reducing as for Weyl to Maxwell's equations, also 4 more equations obtained under variation of f_i, — they should be those additional equations to Maxwell's.

10. I undertook the calculations analogous to those developed in Klein's paper in the *Gött. Nachs. 1918* and by Weyl on page 261 and further in the 4-th edition of his book. I've obtained the system of conditions that the world function M should satisfy in order that $J = \int M dx_1 dx_2 dx_3 dx_4$ be coordinate and scale invariant.

11. Amongst the conditions in item 10 there are four equations of the form that is necessary for the generalized law of energy conservation, and also four equations that have, as in Weyl's theory, the form of Maxwell's equations. I did not examine this system in more detail, and that is why I don't know what results could be obtained from the equations that are additional to Maxwell's.

12. Since I know electrodynamics rather badly, four of us decided to collectively apply the items 10 and 11 to a particular world function: Bursian, Krutkov, Frederix, and myself as a calculator. In general our program is as follows: 1) ascertain all equations following from the world function M, 2) apply these equations to the case of the electron at rest, i.e. find the solution of those equations for the spherical symmetry, and 3) study the obtained solution.

13. Schouten's paper is extremely interesting, but he is following an approach different from mine, as, of course, you understand from my paper. He obtains the generalizations using the analytical method [e.g. item 4, eq. III, page 63 of Schouten's paper] and then restricting the tensorial parameters $\Gamma^i_{\nu\mu}$ and $\Gamma'^i_{\nu\mu}$ (in his notations). I am looking for a generalization using geometrical intuition, but as opposed to Weyl, I populate the space not only by curved lines (one can compare Weyl's infinitesimal *Verpflanzung*), but also by curved hypersurfaces (all my procedures are with contragradient vectors). For these reasons, I think that it would be still possible to submit my paper translated to the German language; it is already translated and the other day I shall send it to you with a request to pass it to Schouten.

Dear Pavel Sigizmundovich, I beg you to express your opinion (even briefly) on what I have written above as well as on the possibility to publish the paper that was sent to you.

II. On the shape of the universe.

1. From my second note in Russian that I have sent to you, you have seen that under certain assumptions common to those of Einstein and De Sitter it is possible to obtain the universe with the space of a (spatially!) constant curvature, the radius of curvature of which is varying with time.

2. It seems to me essential to relate the shape of the universe with two questions: 1) on the causality and 2) on the centrifugal force. The following will serve as an explanation of this.

3. The causality principle should be understood from general considerations, as it is shown by Hilbert in his *Zweite Mitteilung, die Grundlagen der Physik*. But one can extract a part out of it and express it in the form of the following statement:

   One can always choose from the four coordinates x_i one and only one x_4 such that, firstly, two world points A and B (events) with $x_{4A} < x_{4B}$ can be called cause and effect, and, secondly, for allowed transformations x_i to $\bar{x}_i$ one can always find a coordinate $\bar{x}_j$ such that $\bar{x}_{jA} < x_{jB}$.

   The sense of this statement is that relations between cause and effect are invariant under relativistic transformations of the world coordinates.

4. Item 3 is limiting: 1) the arbitrariness of g_{ik} (law of inertia), 2) the arbitrariness of the world's region in which the events are considered (*Nullkegel*!) and 3) the arbitrariness of the coordinates transformation.

5. The causality principle in the mentioned form enables us to separate space from time and to define in general terms the character of g_{ik} or ds^2 by dividing ds^2 into the spatial part $\sum_{i,j,k=1,2,3} g_{ik}dx_i dx_k$, the mixed part $\sum_{i=1}^{3} 2g_{i4}dx_i dx_4$, and the temporal part $g_{44}dx_4^2$.

6. The general form of the spatial part should somehow depend on the centrifugal force issue. The space is filled (on the average) uniformly by matter. Under such assumption the centrifugal force principle could be formulated as follows

7. If the infinitesimal material part of space is rotating around a certain axis (one must determine the rotation in the curved space!) then the apparent centrifugal force will be the same as in the case when the point is immobile, but the whole space is moving correspondingly (one has to define this motion very carefully!).

8. The gravitational potentials g_{ik}, density ρ and the Λ value (*Kosmologisches Glied*) should be found in such a way that the causality and centrifugal force principles would be satisfied.

9. I have read Thirring's paper and become very curious about it, but I was not quite satisfied by it; much can be argued from the mathematical side against Thirring's calculations.

I will write to Thirring and will ask him to send me a print of his article, but I fear he will disregard my small request. Maybe, having an occasion, you could ask him to send me the works, pointing out that they won't lay idly at my place. I am, though, ashamed to burden you with that, so I do not hope that my request would be fulfilled.

The considerations of the 2nd section are just fantasies so far, and it is not worth for you to react on those. If I succeed with anything in that direction, I will write to you again.

`hand written`

Dear Pavel Sigizmundovich,

I know that you are very busy, but still would you please send me a brief note whether I may send you letters (written using a typewriter) similar to this one.

Send my regards to Tatyana Alekseevna. If you decide at any time to come to Petrograd (now it's easy to do as a leisure trip), then of course my apartment is always available to you, as well as me myself.

Sincerely yours,

A. Friedmann.

My mailing address: Petrograd, Main Physics Observatory, Vasilyevsky island, 23 line, house 2. I live at Vasilyevsky island line 5, house 36, apt 13.