3.2.4 Quantum simulation

Quantum simulation represents, along with Shor’s and Grover’s algorithms, one of the three main experimental applications of quantum computers. Of the three, quantum simulation is in fact the application of quantum computers that has actually been used to solve problems that are apparently too difficult for classical computers to solve. As larger-scale quantum computers are developed over the next five and ten years, quantum simulation is likely to continue to be the application for which quantum computers can give substantial improvements over classical computation.

Quantum simulation was in fact the first proposed application for which quantum computers might give an exponential enhancement over classical computation. In 1982, Feynman noted that simulating quantum dynamics on a classical computer was apparently intrinsically hard. Merely to write down the state of a quantum system made up of N two-state systems such as spins took up exponential amounts of space in the memory of a classical computer; and determining the dynamical evolution of such a state required the multiplication of exponentially large matrices. Suppose, Feynman continued, that it were possible to construct a “universal quantum simulator”, an intrinsically quantum device whose state and dynamical evolution could be programmed to mimic the behavior of the quantum system of interest. Such a device, he concluded, could function as a quantum “analog” computer, capable of reproducing the behavior of any desired quantum system.

Feynman merely noted the potential existence of such universal quantum simulators: he did not supply any prescription for how such a universal quantum analog computer might be realized in practice. In 1996, however, Lloyd, Wiesner, and Zalka showed that conventional “digital”
quantum computers could be programmed to perform universal quantum simulation. Since then, Cory et al. have used room-temperature nuclear magnetic resonance (NMR) QIPs to perform coherent quantum simulations of harmonic oscillators [81,82,83] and chaotic quantum dynamics such as the quantum Baker’s map [84,85]. Note that for the purpose of quantum simulation, the apparent lack of scalability of a room-temperature NMR QIP does not prevent such a processor from supplying an apparently exponential speed-up over a classical computer: simulating high-temperature quantum systems is still apparently exponentially hard [86].

An example of a large-scale experimental realization of quantum simulation is the use of solid-state NMR QIPs to study the diffusive limit of transport of dipolar coupled spins in dielectric single crystals. The multibody dynamics were studied over times of tens of seconds, corresponding to of order $10^6$ times the spin-spin correlation time, and spin transport over a distance of 1 µm. One result of these studies was to reveal that the diffusion constant for the two-spin dipolar ordered state is roughly 4 times faster than that of the single-spin, Zeeman ordered state. This speedup was not predicted by theoretical models and has been attributed to constructive interference in the transport of the two-spin state. Today solid-state NMR permits selected multibody problems to be addressed, the field does not yet have sufficient control to enable universal quantum simulation [87,88].

Another potentially interesting source of problems relevant to the sciences are continuous, numerical problems such as integration and Feynman integrals. Because Grover’s algorithm gives a quadratic speedup for not just search but also counting, it can be applied to get a quadratic speedup for integration in a natural way [14]. It remains an interesting open question whether some of the more sophisticated quantum walk techniques or other quantum algorithm techniques can be used in this context.

At the other end of the spectrum, QIT has provided novel algorithms for classically simulating quantum systems with limited entanglement. Vidal et al. [89] characterized the scaling properties of the ground-state entanglement in several 1-D spin-chain models both near and at the quantum-critical regimes. They showed that the entanglement length scales logarithmically in the number of spins [it scales like log(L)]. Vidal [15] recently gave an efficient classical algorithm for simulating the dynamics of 1-D spin chains that runs in time exponential in the entanglement length. Experimental results suggest that this method may be very effective in simulating a variety of systems. Extension of these results to 2-D and 3-D would be very interesting.


