Answers to the Exam Quantum Theory, 12 January 2026

each item gives 2 points for a fully correct answer, grade = total $\times\, 9/24 + 1$

1. *(a)* $dE_n/d\lambda = \langle\psi_n|dH/d\lambda|\psi_n\rangle + \langle\psi_n|H|d\psi_n/d\lambda\rangle + \langle|\psi_n/d\lambda|H|\psi_n\rangle = \langle\psi_n|dH/d\lambda|\psi_n\rangle +$ $E_n(d/d\lambda)\langle\psi_n|\psi_n\rangle = \langle\psi_n|dH/d\lambda|\psi_n\rangle$.
   (b) The velocity operator is $v = dH/dp$, so $\langle v\rangle = \langle dH/dp\rangle = dE/dp$.
   (c) $dE_n/dm = (-1/m)\langle\psi_n|T|\psi_n\rangle + (1/m)\langle\psi_n|V|\psi_n\rangle$; since $dE_n/dm = 0$ it follows that $\langle\psi_n|T|\psi_n\rangle = \langle\psi_n|V|\psi_n\rangle$.

2. *(a)* $-i\hbar d/dx = p$ is the momentum operator, which is Hermitian, so $H = H^\dagger$ is also Hermitian.
   (b) The Hamiltonian can be written in terms of Pauli matrices, $H = vp\sigma_x$. Time reversal inverts p and σ_x, leaving H unchanged.
   (c) In terms of the wave number k, we have $H = \hbar v k\sigma_x$, with eigenvalues $E(k) = \pm\hbar v k$.

3. *(a)* Since a contour integral that encircles the solenoid has $\oint \vec{A}\cdot d\vec{l} = \Phi$, it is not possible to find a gauge where $\vec{A} = 0$ outside the solenoid if $\Phi \neq 0$.
   (b) $\Psi(\vec{r}_{\text{screen}}) \propto \sum_{n=1,2} \exp\left(\frac{i}{\hbar}\int_n (m\vec{v} + e\vec{A})\cdot d\vec{l}\right)$, summed over the two paths $n = 1, 2$ that reach the screen through one slit or the other.
   (c) $|\Psi(\vec{r}_{\text{screen}})|^2 = \text{constant} + 2\cos\left(\hbar^{-1}\int_1 (m\vec{v} + e\vec{A})\cdot d\vec{l} - \hbar^{-1}\int_2 (m\vec{v} + e\vec{A})\cdot d\vec{l}\right)$
   The argument of the cosine is a Φ-independent constant plus $(e/\hbar)\oint \vec{A}\cdot d\vec{l} = e\Phi/\hbar$. So $\Delta\Phi = 2\pi\hbar/e = h/e$.

4. *a)* $ae^{\beta a^\dagger}|0\rangle = \sum_{n=0}^{\infty}\frac{1}{n!}\beta^n a(a^\dagger)^n|0\rangle = \sum_{n=0}^{\infty}\frac{1}{n!}\beta^n n(a^\dagger)^{n-1}|0\rangle = \beta e^{\beta a^\dagger}|0\rangle$
   b) $\langle\alpha|\beta\rangle = e^{-|\alpha|^2/2}\langle 0|e^{\alpha^* a}|\beta\rangle = e^{-|\alpha|^2/2}e^{\alpha^*\beta}\langle 0|\beta\rangle = e^{-|\alpha|^2/2-|\beta|^2/2}e^{\alpha^*\beta}$, hence $|\langle\alpha|\beta\rangle|^2 = e^{-|\alpha|^2-|\beta|^2}e^{\alpha^*\beta+\alpha\beta^*} = e^{-|\alpha-\beta|^2}$.
   c) $\bar{n} = \langle\beta|a^\dagger a|\beta\rangle = \beta^*\beta$, $\overline{n^2} = \langle\beta|(a^\dagger a)^2|\beta\rangle = \langle\beta|(a^\dagger)^2 a^2|\beta\rangle + \langle\beta|a^\dagger a|\beta\rangle = (\beta^*\beta)^2 + \beta^*\beta$, so $\operatorname{var} n = \beta^*\beta = \bar{n}$.