Dr. C. H. WIND. Measurements regarding the Sissingh magneto-optical phase difference in the case of polar reflexion from Nickel.

Through Sissingh's experiments ${ }^{1}$ ) concerning equatorial reflexion on iron magnets, attention has been drawn, for the first time, to the fact, that there is a difference between the phase of the magneto-optical component in the KERr effect, as deduced from observation, and that derived from the theory of Lorentz ${ }^{2}$ ), which difference is nearly constant within ample limits of the angle of incidence. Goldhammer ${ }^{3}$ ) has inserted it in his theory. On the other hand Zeeman ${ }^{4}$ ) has shown that the same constant phase difference occurs in the case of polar reflexion on iron too, and that it also exists in polar reflexion on cobalt. My investigation shows, or makes it at least very probable, that this is likewise the case for nickel and consequently confirms the opinion, that Sissingr's phase difference has a physical meaning.
For my experiments the apparatus as composed and

[^0]described by Zeeman ${ }^{1}$ ) was made use of - only a few alterations of minor importance were made.

As to the electro-magnet the core ending in a truncated cone, to the top of which the mirror was attached by means of canadian balm, had a diameter of 12 mM ; a special experimental examination ${ }^{2}$ ), made beforehand, had proved this thickness to be the most desirable. The mirror (a circular little disc, $11 / 2 \mathrm{mM}$. thick, 5 mM . diam.), though no donbt the best among a great number manufactured for the purpose of pure nickel (as it is produced in cubes by Trommsdorff at Erfurt), was not quite flat; with the aid of a microscope there could be discerned scratches, porous spots, and furrows of a more or less regular shape, which to a certain extent suggested a crystalline and fibrous structure of the metal. As long as the surface was unimpaired its optical constants were nearly equal to those given by Drude ${ }^{3}$ ) for pure nickel; during the experiments, however, they were subject to continual changes, which, in some instances, when the mirror was too much heated, became rery considerable. This was what especially occurred during the first series of observations (the angle of incidence $\alpha$ being $39^{\circ} 4^{\prime}$ ); in the second and third series ( $\alpha=55^{\circ}$ and $\alpha=75^{\circ}$ ) the intervals between the separate observations were taken so long, that the temperature of the mirror could not rise higher than $60^{\circ}$,
${ }^{1}$ ) Zeeman. Dissertation. Leiden, 1893 ; Arch. Néerl. 27, p. 252. 1893.
${ }^{2}$ ) cf. Zeeman. Diss., p. 10.
${ }^{3}$ ) Drude. Wied. Ann. 39, p. 522. 1890.
resp. $40^{\circ}$ Celsius, and here the above changes of the optical constants, which were continually watched during the experiments, proved of no importance. As for the rest the mean of the optical constants at the beginning and at the end of the observations was made use of for the calculation of the results, the influence of the changes in the surface of the mirror may be taken to be sufficiently eliminated. In this respect the first series left much to be desired.

In the first and the third series of observations, nulland minimum-rotations were measured at every one of the eight combinations of principal positions of the nicols, which are possible ${ }^{1}$ ), as was in some cases done by Sissingh; in the second series only at four of them, two in which the plane of polarisation was parallel, two in which it was perpendicalar to the plane of incidence. Whether all systematic faults, which may be caused by imperfect condition of the nicols and other parts of the apparatus, are really quite eliminated, even when the observations are made at the eight combinations mentioned above, is a quaestion, which cannot be taken as to have entirely been settled by Sissingh's examination on this head ${ }^{2}$ ). I should rather think a very elaborate investigation to be required in order to state the degree, in which this elimination may be considered as to be attained by the method of observation.

All the measurements have been made with yellow

[^1]light (wave-lengths from 564 to $614 \mu \quad \mu)$. The strength of the magnetic field was determined by the same method as applied by Zeeman ${ }^{1}$ ). To derive from this the intensity of magnetization of the mirror, a special magnetic examination of the nickel used would have been necessary, for which there was at the moment no opportunity and no timë. With the known data regarding the magnetie qualities of nickel, it would ensue from my observations, that the way, in which the amplitude of the magnetic component in the KERR effect depends upon the angle of incidence, is not correctly described by the theories of Lorentz or Goldhammer. It may be mentioned here, that the measurements of Sissingil and Zeeman seem to show a deviation from these theories in the same sense as mine. As, however, the determination of the intensity of magnetization is no easy matter and becomes especially very incertain for nickel, this deviation must not be considered of much importance for the present, though we might feel inclined to refer it to the crystalline structure of the metal, suggested above. If it had been my intention to compare theory and experiments with respect to the amplitude of the magnetic component, I should have preferred leaving the position of the msubmagnet," as well as the current power in the magnetic coil unaltered at different angles of incidence, in order to avoid the difficulties just mentioned as much as possible. However, at this examination as at Zeeman's the Sissingh phase dilference was prominent, and to

[^2]determine it no knowledge of proportions of magnetisations at different angles of incidence is required, seeing that the value of this difference is deducted from the proportion of rotations at one and the same $\alpha$. In order to determine this proportion as minutely as possible the magnetization at each angle of incidence should be raised as much as possible. This end could be reached only in a different degree for each separate angle of incidence, seeing that when this angle decreased, the distance of the sub-magnet, that is the width of the air-gap in the magnetic circuit, had to be made larger. Though in this way comparison of the amplitudes was rendered rather valueless, it had to be sacrified, in order that greater precision for the determination of the phase might be acquired.

In communicating my measurements 1 make use of the notations, as employed by Sissingh and Zeeman ').
I.

Angle of incidence $\alpha=39^{\circ} 4^{\prime}$; Strength of magnetic field 2190 C.G.S.:

$$
\begin{aligned}
& \psi^{\circ}{ }_{l p}=1^{\prime}, 96 \pm 0,22 ; \quad \psi^{0}{ }_{l /}=-10^{\prime}, 11 \pm 0,35^{5} \\
& \psi^{\circ}{ }_{i,}=0^{\prime}, 50 \pm 0,15 ; \quad \psi^{\circ}{ }_{i \prime}=12^{\prime}, 90 \pm 0,16 \\
& \psi^{\prime \prime \prime}{ }_{l p}=9^{\prime}, 95 \pm 0,21 ; \psi^{\prime \prime \prime}{ }_{l / \prime}=-12^{\prime}, 50 \pm 0,23^{5} \\
& \psi^{\prime \prime \prime}{ }_{i, 1}=-9^{\prime} 88, \pm 0,18^{5} ; \psi^{\prime \prime \prime}{ }_{i p}=12^{\prime}, 31 \pm 0,30
\end{aligned}
$$

[^3]Mean values:

$$
\begin{aligned}
& \left.\psi_{i,}^{0}=-0,73^{1}\right) \pm 0,13^{5} ; \quad \psi^{\circ}{ }_{i \prime \prime}=11^{\prime}, 25 \pm 0,19^{5} ; \\
& \psi^{m_{i, 1}}=-9^{\prime}, 91 \pm 0,14 ; \quad \psi^{\prime \prime \prime}{ }_{i, \prime}=12^{\prime}, 40+0,19 .
\end{aligned}
$$

Before Kerr-observations:
Principal angle incidence $I=75^{\circ} 32^{\prime}$,
Principal azimut $H=31^{\circ} 26^{\prime}$;
After Kerr-observations: $I=73^{\circ} 13^{\prime}, H=34^{\circ} 9^{\prime}$.
The formulae for the calculation of $m_{i}$ become:

$$
\begin{aligned}
& \text { tg. } m^{\prime \prime \prime}{ }_{i}=-3.589-3,274^{\frac{\psi^{\prime \prime \prime}}{\psi_{i p}}} \\
& \text { cotg. } \mathrm{m}_{i \omega}^{\circ}=3,589-4,238 \frac{\psi^{\circ}{ }_{i a}^{\circ}}{\psi_{i p}^{\circ}}
\end{aligned}
$$

So we derive from the $\psi^{m \prime}: m^{m \prime \prime}=26^{\circ} 56^{\prime} \pm 3^{\circ} 52^{\prime}$,

$$
10^{3} \mu^{m m_{i}}=-1,339 \pm 0.37^{5}
$$

and from the $\psi^{\circ}: m^{\circ}{ }_{i}=14^{\circ} 30^{\prime}, 5 \pm 1 \mathrm{I}^{\prime}$,

$$
10^{3} \mu_{i}^{0}=-1,276 \pm 0,023^{5}
$$

Most probable values, as resulting from this series of observations:

$$
m_{i}=14^{\circ} 32^{\prime} \pm 11^{\prime}, 10^{3} \mu_{i}=-1,293^{5} \pm 0,020
$$

The circumstance that so little weight is to be attached to the final values resulting from the minimum-rotations agrees with the fact, that a variation of only $0^{\prime}, 8$ in

[^4]the value of one of the minimum-rotations would transfer $m^{\prime \prime \prime}{ }_{i}$ into $m^{\circ}$. Besides this, on recently developed grounds the null-rotations should be preferred to determine the phase.

Calculated after Prof. Lorentz's theory:

$$
m_{i}=-23^{\circ} 30^{\prime}, 5
$$

## II.

$\alpha=55^{\circ}$, Strength of magnetic field 9560 C.G.S.

$$
\psi^{\circ}{ }_{l,}=5^{\prime}, 37 \pm 0,26 ; \psi^{\psi_{l a}}=-7^{\prime}, 39 \pm 0,46
$$

$$
\psi_{i u}=-5^{\prime}, 35^{5} \pm 0,32^{5} ; \psi_{i p}^{0}=7^{\prime}, 78^{5} \pm 0,32^{5}
$$

$$
\psi^{\prime \prime \prime} l_{p}=10^{\prime}, 09 \pm 0,23 ; \psi_{l a}^{m_{l a}}=-14^{\prime}, 54 \pm 0,31^{5}
$$

$$
\psi^{m}{ }_{i a}=10^{\prime}, 48^{5} \pm 0,23^{5} ; \psi^{m i}{ }_{i n}=13^{\prime}, 22 \pm 0,20
$$

Mean values:

$$
\begin{aligned}
& \psi_{i a}=-5^{\prime}, 36 \pm 5,21^{5} ; \psi_{i p}=7^{\prime} 59 \pm 0.29 \\
& \psi^{m \prime \prime}=-10^{\prime}, 29 \pm 0,17 ; \psi_{i \mu}^{m}=13^{\prime} 88 \pm 0,19^{5}
\end{aligned}
$$

Mean values: $I=74^{\circ} 34^{\prime}, 5 ; H=31^{\circ} 53^{\prime}$.
The formulae for the calculation of $m_{i}$ become:

$$
\begin{aligned}
& \operatorname{tg} \cdot m_{i}^{m_{i}}=-1,470-1,348 \frac{\psi^{m_{i j}}}{\psi^{m_{i a}}} \\
& \operatorname{cotg} . m_{i}^{0}=1,470-2,351 \frac{\psi^{0_{i, ~}}}{\psi^{{ }_{i i \prime}}}
\end{aligned}
$$

Derived from the $\psi^{m}$ :

$$
m^{m}=19^{\circ} 0^{\prime}, 5 \pm 2^{\circ} 1^{\prime} ; 10^{3} \mu^{m}=-1,368 \pm 0,014^{5}
$$

Derived from the $\psi^{\circ}$ :

$$
m_{i}-17^{\circ} 43^{\prime} \pm 28^{\prime}, 5 ; 10^{3} \mu_{i}=1,333^{\mathrm{s}} \pm 0,035
$$

Most probable values, derived from the observations:

$$
m_{i}=17^{\circ} 47^{\prime} \pm 28^{\prime} ; 10^{3} \mu_{i}=-1,359 \pm 0,013
$$

Calculated after the theory :

$$
m_{i}=-18^{0} 36^{\prime}
$$

III.
$\alpha=75^{\circ}$, Strength of magnetic field 12470 C.G.S.

$$
\begin{aligned}
& \psi^{0}{ }_{1 p}=6^{\prime}, 16 \pm 0,13 ; \psi^{0}{ }_{l a}=-6^{\prime}, 44^{5} \pm 0,19^{5} \\
& \psi_{i a}^{0}=-6^{\prime}, 09 \pm 0,19^{5} ; \psi_{i p}=6^{\prime}, 61 \pm 0,15^{5} \\
& \psi^{m}{ }_{l p}=6^{\prime}, 25 \pm 0,15 ; \psi^{m_{l a}}=-8^{\prime}, 12 \pm 0,25^{5} \\
& \psi^{\prime \prime{ }_{i}}=-6^{\prime}, 28 \pm 0,15 ; \psi_{m_{i p}}=8^{\prime}, 01 \pm 0,27^{5}
\end{aligned}
$$

Mean values:

$$
\begin{aligned}
& \psi_{i a}^{o}=-6^{\prime}, 12 \pm 0,12 ; \psi_{i_{p}}=6^{\prime}, 53 \pm 0,12^{5} \\
& \psi^{m_{i a}}=-6^{\prime}, 26^{5} \pm 0,10^{5} ; \psi^{\prime{ }^{\prime \prime}}=8^{\prime}, 07 \pm 0,19
\end{aligned}
$$

Mean values: $1=75^{\circ} 58^{\prime} ; H=30^{\circ} 57^{\prime}, 5$.
The formulae for the calculation of $m_{i}$ become:

$$
\begin{aligned}
& \operatorname{tg} . m^{m_{i}}=-0,0881-0,6038 \frac{\psi^{m i} t p}{\psi^{\prime m}}, \\
& \operatorname{cotg} . m_{i a}^{o}=0,0881-1,669 \frac{\psi^{o}{ }_{i a}}{\psi_{i p}^{o}}
\end{aligned}
$$

Derived from the $\psi^{m}$ :

$$
m^{m}=34^{\circ} 34^{\prime} \pm 50^{\prime} ; 10^{3} \mu^{m} i_{i}=-1,039^{5} \pm 0,014^{5}
$$

Derived from the $\psi^{n}$ :

$$
m_{i}^{\prime \prime}=3 \cdot 1^{0} 10 \pm 37^{\prime}, 5 ; 10^{3} \mu_{i}^{u_{i}}=-1,031-0,016
$$

Most probable values, derived from the observations. -

$$
m_{i}=32^{0} 24^{\prime}, 5 \pm 30^{\prime} ; 10^{3} \mu_{i}=-1,035^{5} \pm 0,011
$$

Calculated after the theory:

$$
m_{i}=-4^{\circ} 44^{\prime}
$$

Resuming the results we have:

| $\begin{array}{c}\text { Angle of } \\ \text { incidence. }\end{array}$ | $m$ duduced from |  |
| :---: | :---: | :---: | :---: |
| observations |  |  |$\left.\quad \begin{array}{c}\text { theory. }\end{array}\right)$

Taking into consideration that the observations at $39^{\circ} 41^{\prime}$ have been influenced by considerable variations of the optical constants and that for that reason the values taken into account for them are certainly not exact, there are sufficient grounds to conclude from the above measurements to the probability of the existence of a Sissingh phase difference, which is nearly constant within ample limits for the angle of incidence. The experiments agree so much the better with the assumption of a constant phase difference as in the method of determining the optical constants (calculating them according to Drude's approximating formulae from $\phi$ and $h$ measured at an angle of incidence not very much differring from I) the theoretically calculated phase may easily deviate $10^{\prime}$ from the value which
would have been found with the aid of correct optical constants, and the coefficients in the equations, which serve to derive $\mu$ and $m$ from the rotations observed, , are likewise not very exact.

If in determining the numerical value of the Sissingh phase difference we don't take into account the results of the observations at $39^{\circ} 4^{\prime}$, which are by far the least reliable, also because the "Sub-magnet" was not so well centred during these experiments as during the other series, we find:

$$
S_{X i}=36^{\circ} 44^{\prime} \pm 20,^{\prime} 5 \text { for } D \text {-light, }
$$

whereas Zeeman had derived a preliminary value of $30^{\circ}$ from measurements on a nickel mirror, electrolytically plated on Kundt's platined glass. The most probable error given at the value of $S_{N i}$, as only calculated from the single orientations of the Nicols, cannot give an exact measure for the accuracy of the results obtained. By reason of the sources of error mentioned above, the degree of accuracy obtained is somewhat less.

The conclusions, which Zeeman ${ }^{~}$ ) derived from the numerical value of $S_{N i}$ after his preliminary determination, hold good at the value more carefully determined by me.

[^5]
[^0]:    $\left.{ }^{1}\right)$ Sissinah, Phil. Mag. 1891 ; Arch. Néerl. 27. 1893.
    ${ }^{2}$ ) Lorentz. Versl. en Meded. Kon. Akad. v. Wetensch., Amst.
    II, 19; Arch. Néerl., 19. Cf. van Loghem, Dissertation, Leiden.
    ${ }^{3}$ ) Goldhammer. Wied. Ann. 46, p. 71, 1892.
    $\left.{ }^{4}\right)$ Vid. Communications Nr. 5, 8, 10, and citation 1) p. 3.

[^1]:    ${ }^{1}$ ) Sissingh. l. c. ; Kaz. Dissertation. Amsterdam, 1884.
    ${ }^{2}$ ) Sissingh. Dissertation. Leiden, 1885.

[^2]:    ') Zeeman. I. c.

[^3]:    ${ }^{1}$ ) Sissingin. l. c.; Zeman. l. c.

[^4]:    $\left.{ }^{1}\right) \psi^{\circ}{ }^{\prime} p$ being positive the sign of the mean value of $\psi^{\circ}{ }_{i a}$ should be, according to the relation $\psi_{i a}=-\psi_{1 \prime}$ (Zeeman. Arch. Néerl., p. 268, 286. 1893).

[^5]:    ${ }^{1}$ ) Zeman. Arch. Néerl., 27, p. 296. 1893. Vid. also: Coinmunications 5, 8 and 10.

