

# **Non-decoupling of heavy scalars in cosmology**

**P R O E F S C H R I F T**

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The background image is the Hubble ultra deep field, an image of an estimated 10,000 galaxies in the constellation Fornax. These galaxies were formed from the density perturbations that were created during inflation and are best studied from the cosmic microwave background radiation. The image of this radiation is used as a background for the letters on the cover. Finally, a curved inflaton trajectory is depicted, which will make features in the power spectrum that might be possible to study using late time cosmology.

'Lieve hart mijn boek is af, mijn boek is af!' - Multatuli



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# CHAPTER 1

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## Introduction

### 1.1 The standard model of cosmology

For a long time it has been realised that the observed darkness of the night contradicts an eternal infinite universe. The first known account of the dark night paradox, most commonly known as Olbers' paradox (Olbers, 1823), is credited to Digges (Digges, 1576) and Kepler (Kepler, 1610). Later accounts include work by Halley (Halley, 1720a,b) and Cheseaux (Cheseaux, 1744). The first attempt on a resolution to this problem is by Lord Kelvin (Kelvin, 1901). For a historic account including translated reprints of the cited articles, see Harrison (1987).

In the following years, progress was much more swift. In 1915, Einstein published his papers concerning the theory of general relativity (Einstein, 1915, 1916), which point to a universe that is unstable against collapse, and thus must either be expanding at a declining rate or collapsing at an increasing rate. Einstein perceived this as a problem — he assumed the universe was stationary — and introduced a cosmological constant to solve this (Einstein, 1917). However, a cosmological constant still does not allow for a stable stationary universe, as shown by de Sitter in 1918. In two papers, de Sitter analysed the behaviour of empty universes and showed that any of those universes only has unstable fixed points (de Sitter, 1918a,b).

In the years following, Friedmann (1922), Lemaître (1927)<sup>1</sup>, Robertson (1929) and Walker (1933) wrote down a maximally symmetric metric that does allow for

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<sup>1</sup>English translation: Lemaître (1931)

expansion or collapse by an overall scale factor  $a(t)$ ,

$$ds^2 = dt^2 + a(t)^2 d\mathbf{x}^2, \quad (1.1)$$

and used Einstein's field equations to solve for the evolution of  $a(t)$ ,

$$H^2 = \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho(t) - \frac{kc^2}{a^2} + \frac{\Lambda c^2}{3}, \quad (1.2)$$

where  $c$  is the speed of light and  $k$  is a parameter that describes the spatial curvature of the universe,  $k = 0, \pm 1$  for a flat, positive or negatively curved spatial hypersurface. Furthermore,  $\Lambda$  parametrises the cosmological constant Einstein introduced to allow stable cosmological solutions with matter and  $\rho$  describes the energy density. In this equation the Hubble parameter  $H = \dot{a}/a$  is introduced, which is the rate of expansion of the universe. Its current best estimates include  $70.6 \pm 3.1$  (km/s)/Mpc (Suyu et al., 2010) and  $70.4^{+1.3}_{-1.4}$  (km/s)/Mpc (Jarosik et al., 2010). The Hubble parameter has the units of inverse time,  $H^{-1}$  is therefore a timescale, the Hubble time, which is approximately the age of the universe. Multiplying with the speed of light gives the Hubble radius  $cH^{-1}$ , which is the maximum distance anything can have travelled in a Hubble time.

The evolution of  $\rho$  can be found from the second Friedmann equation

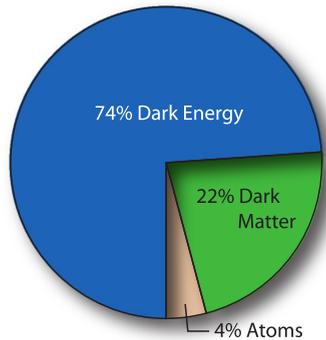
$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}\left(\rho + \frac{3p}{c^2}\right) + \frac{\Lambda c^2}{3} \quad (1.3)$$

together with an equation of state  $p = f(\rho)$  that relates the energy density to a pressure  $p$ . We will take a linear relation  $p = w\rho$ , with  $w$  an arbitrary parameter, as this suffices to describe the cases of interest in this chapter. In fact, with this linear relation also curvature and the cosmological constant can be defined in terms of an effective energy density parameter  $\rho$  and effective pressure  $p$ , see table 1.1. Then all the contributions  $\rho_{\text{matter}}$ ,  $\rho_{\text{radiation}}$ ,  $\rho_k$  and  $\rho_\Lambda$  have to sum to  $3H^2/(3\pi G) = \rho_c$ , which allows one to write the Friedmann equation (eq. 1.2) as

$$1 = \sum_i \frac{\rho_i}{\rho_c} \quad (1.4)$$

where  $i$  labels the different components. The first experimental evidence of a non-stationary universe came in 1929, when Hubble showed that almost all galaxies are redshifted and thus move away from us, and that the redshift is positively correlated with the distance of the galaxy. Much more convincing evidence came in 1965 when Penzias and Wilson found the Cosmic Microwave Background. This background

was already predicted by Gamow (1948a,b) and Alpher and Herman (1948) and is created in the early universe, about 380,000 years after the big bang. In an expanding universe, temperatures are higher in the past, and before 380,000 years after the big bang the temperatures were too high for neutral hydrogen to exist. As photons are very effectively scattered by free electrons, the mean free path of a photon was much shorter than the Hubble radius,  $cH^{-1}$ . Around 380,000 years after the big bang, the recombination time, the universe cooled down enough for neutral hydrogen to form. Neutral hydrogen scatters photons much less efficient, so suddenly the mean free path of photons became much larger than the Hubble radius. As a result, the universe became approximately transparent for photons. These photons, redshifted by a factor  $T_{\text{now}}/T_{\text{formation}} \sim 1100$ , now form the 2.73 K microwave background. Due to the finite lifetime of our universe, not all lines of sight do end on stars. Yet, in an expanding universe of nonzero temperature one expects an all-sky background from the recombination surface. In contrast to the all-sky stellar radiation from an eternal universe, this background exists and thus provides strong evidence for an expanding universe.



**Figure 1.1:** The current measurements on the energy density components of our universe (image courtesy: LAMBDA, NASA.)

Furthermore, from (eq. 1.3) it is clear that the rate of expansion of the universe should decrease ( $\ddot{a} < 0$ ) for energy density components with  $w > -1/3$ , while it increases when  $w < -1/3$ . Recent evidence (Perlmutter et al., 1998, 1999, Riess et al., 1998, Spergel et al., 2003, Komatsu et al., 2010) shows the Hubble parameter  $H$  is currently increasing, which means that a cosmological constant or some other form of dark energy must actually give a large contribution to the total energy density.

Cold matter	$w = 0$
Radiation	$w = 1/3$
Curvature	$w = -1/3$
Cosmological constant	$w = -1$

**Table 1.1:** Equation of state for the different components of energy density.

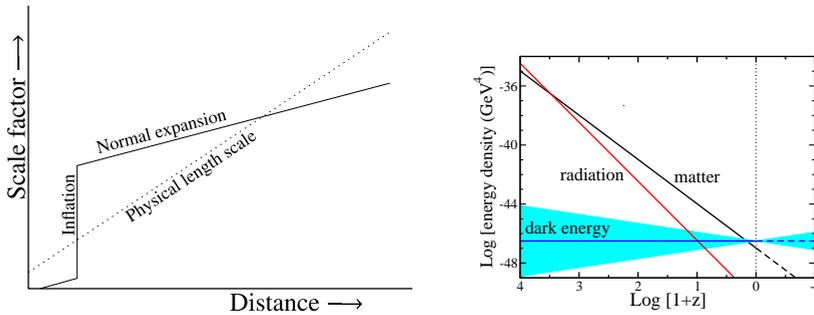
Besides dark energy, there is also a missing energy component that satisfies the matter equation of state, dark matter. From a wide range in scales it is known that normal matter cannot explain the observed gravitational fields. At small scales rotation curves in galaxies (Rubin and Ford, 1970) do not match the visible matter distribution. At intermediate scales, peculiar motion in clusters (Zwicky, 1933) and gravitational lensing (see eg. Bacon et al., 2000, Refregier, 2003 for constraints from weak lensing from the large scale structure) point to a mass component that is not visible. At large scales, the formation of the large scale structure (see Springel et al., 2005 for an overview of the simulations, Tegmark et al., 2006, Reid et al., 2010 for measurements) and the observations of the Cosmic Microwave background (Spergel et al., 2003, Komatsu et al., 2010) need a dark matter component to understand the data. We know that this dark matter component cannot be baryonic, as the baryonic matter component can be determined via primordial nucleosynthesis (Kernan and Krauss, 1994, Copi et al., 1995, Coc et al., 2004). Furthermore, modified gravity with baryonic matter does not fit lensing observations of the Bullet cluster (Clowe et al., 2004) and the Cosmic Microwave Background data (Komatsu et al., 2010, for a nice theoretical discussion see Mukhanov, 2004). These observations point to an invisible component that must have weak interactions with all matter in the universe<sup>2</sup>, and certainly does not interact with light. Combining the current knowledge about energy density components, we find that the universe is, within experimental bounds, spatially flat (Komatsu et al., 2010) and dominated by dark matter and dark energy (see figure 1.1).

In the Friedmann equations (eqs. 1.2 and 1.3) above the evolution of the universe is such that, for  $w > -1/3$ , the Hubble radius is growing faster than the distance between causally connected points, that is points that have been in causal contact. Our universe is currently dark energy dominated, but back in time it certainly was dominated by matter and earlier radiation (figure 1.2(b)). Current observations of

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<sup>2</sup>This does not mean that dark matter is charged with respect to the the weak interaction, although a very popular candidate, the “Weakly Interacting Massive Particle”, is.

the Cosmic Microwave Background show correlated perturbations up to the size of the visible universe. The Hubble horizon at the time of decoupling, and thus the maximum distance between causally connected events, is 380,000 light years, which corresponds to about one degree on the current sky (approximately the first peak in the angular power spectrum, figure 1.3). This means that the Microwave Background consists of approximately 40,000 patches that were causally disconnected at the time of decoupling. Furthermore, the current measured flatness, isotropy and homogeneity of the universe (Spergel et al., 2003, Komatsu et al., 2010) also violates causality or requires large amounts of finetuning, while the lack of phase transition defects (eg. magnetic monopoles) is unnatural. All these problems can be solved by introducing a period of inflation, as proposed by Guth in 1980, which will be the topic of the next paragraph.<sup>3</sup>



(a) Evolution of Hubble radius (solid line) and a physical distance (dotted line), such as the separation between two points.

(b) Evolution of the different energy density components as a function of redshift (image from Frieman et al., 2008).

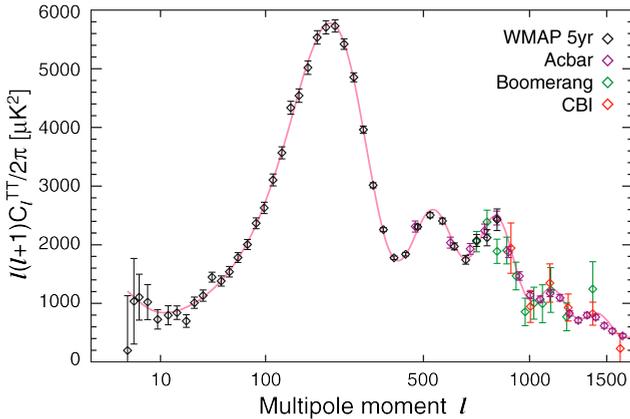
Figure 1.2:

## 1.2 The inflationary paradigm

The solution proposed in Guth (1981) is to use the observation that for energy contributions with an equation of state parameter  $w < -1/3$  the Hubble radius grows slower than the distance between causally connected points. If a period with  $w < -1/3$ ,

<sup>3</sup>Isotropy is actually not solved by inflation, as different dimensions could inflate at different rates. String theory compactifications, discussed in section 1.3.1, actually need anisotropic inflation, as isotropic inflation would also inflate the hidden dimensions.

called an inflationary period, lasts only temporarily, it allows a region that is causally connected before the onset of inflation to cover a causally disconnected region after the inflationary period. When inflation lasts long enough, our current observable universe fits entirely within a region that was causally connected before inflation occurred (figure 1.2(a)). Thus, inflation can solve finetuning problems concerning the homogeneity of the visible universe. Furthermore, during the period of inflation the accelerated expansion reverses the growth of curvature and dilutes matter, thus solving the flatness problem and the issues regarding the absence of cosmological defects. A negative equation of state can be created by a field that has a small kinetic energy term and a large potential energy term. For  $w = -1$  the energy density per unit volume is constant when the universe expands. A vacuum energy, generated by a nonzero vacuum expectation value of a scalar field, is so far the only field theoretical mechanism that can provide an equation of state with a negative  $w$ . The model proposed by Guth used quantum tunneling from an inflating state to the current vacuum to end inflation. Later work by Linde (1982) and Albrecht and Steinhardt (1982) found a new inflation paradigm that used a continuous potential along which the inflaton slowly rolls down.



**Figure 1.3:** Power spectrum of the angular cross correlation function of the temperature of the Cosmic Microwave Background. The red line is the best fit, based on WMAP-data alone, to the  $\Lambda$ CDM model (Dunkley et al., 2009). This fit agrees very well with the higher multipole data. Image from Nolta et al. (2009).

In 1981 Mukhanov and Chibisov showed that perturbations of the metric and the inflaton field lead to perturbations with a power spectrum that depends on the po-

tential of the inflaton field (see also Hawking, 1982, Starobinsky, 1982, Guth and Pi, 1982, Bardeen et al., 1983, Mukhanov, 1985). The power spectrum should be exactly scale invariant for  $w = -1$ , but in order to explain why inflation could end we need an inflaton field that evolves to lower energies. Therefore, a slightly red-tilted power spectrum is predicted. In 1991 the COBE satellite (Smoot et al., 1992) first observed these perturbations. In 1999, the ground based Toco experiment (Torbet et al., 1999) found evidence for the first acoustic peak, later confirmed by the BOOMERanG (Melchiorri et al., 2000) and MAXIMA (Hanany et al., 2000) balloon experiments. In 2001, the quality of the data improved significantly with data from the WMAP satellite (Spergel et al., 2003). Later, the WMAP 5-year data Hinshaw et al. (2009), together with data from balloon experiments (ACBAR (Reichardt et al., 2009), BOOMERanG (Jones et al., 2006) and CBI Readhead et al. (2004)) provided an accurate map of the perturbations to small angular scales (figure 1.3), and confirmed the red tilt of the power spectrum (Dunkley et al., 2009). The current best estimate is from the WMAP 7-year data (Komatsu et al., 2010), which is compatible with an approximately power law power spectrum  $P(k) \propto k^{n_s-1}$ , with  $n_s = 0.963 \pm 0.012$  at 68% CL. Upcoming improved measurements are expected from the ground based Atacama Cosmology Telescope (see Fowler et al., 2010 for the first results) and the Planck satellite (Planck collaboration, 2006), whose results are expected late 2012 or early 2013.

In order to solve the problems that inflation was invented for, inflation needs to last for at least  $55 H^{-1}$ , in which the universe thus expanded by  $e^{55}$  orders of magnitude or 55  $e$ -folds. In order to have so many  $e$ -folds the potential energy is allowed to vary only very slowly. Being consistent with a slightly red tilted power spectrum, the most common mechanism to generate such a slow variation is called slow-roll inflation. Non-inflationary mechanisms that are compatible with the observations of the power spectrum include gauge/cosmology duality (McFadden and Skenderis, 2010a,b), the ekpyrotic scenario (Khoury et al., 2002, Steinhardt and Turok, 2001, 2002) or string gas cosmological models (Brandenberger and Vafa, 1989, Brandenberger et al., 2004). Also, an entirely causal mechanism has been proposed (Turok, 1996).

### 1.2.1 Slow-roll inflation

Single field slow-roll inflation successfully accounts for many of the observed properties of the cosmic microwave background (CMB), including the near scale invariance of the power spectrum of the primordial density fluctuations that seed the observed CMB anisotropies (Mukhanov and Chibisov, 1981). Slow-roll inflation in its sim-

plest form (Linde, 1982, Albrecht and Steinhardt, 1982) still marginally fits the current observational bounds (Komatsu et al., 2010, Larson et al., 2010). Furthermore, all inflation models developed since these earliest models depend on the slow-roll approximation (Steinhardt and Turner, 1984, Salopek and Bond, 1990, Liddle and Lyth, 1992. See Lyth and Liddle, 2009 for a recent text-book discussion) that the Hubble rate is varying only slowly. In terms of the Hamilton-Jacobi slow-roll variables the requirement is that

$$\begin{aligned}\epsilon_H &= -\frac{\dot{H}}{H^2} \ll 1 \quad \text{and} \\ \eta_H &= -\frac{\ddot{\phi}_0}{H\dot{\phi}_0} \ll 1 ,\end{aligned}\tag{1.5}$$

where  $\dot{\phi}_0$  is the rate of change of the inflaton field. In a single-field inflationary scenario, one can also define the slow-roll parameters by requirements on the potential

$$\begin{aligned}\epsilon_V &= \frac{M_{\text{Pl}}^2}{2} \left( \frac{V'}{V} \right)^2 \ll 1 \quad \text{and} \\ \eta_V &= M_{\text{Pl}}^2 \frac{V''}{V} \ll 1 ,\end{aligned}\tag{1.6}$$

which is the more commonly used definition. In the slow-roll limit the parameters (eq. 1.5) and (eq. 1.6) are related as

$$\epsilon_H = \epsilon_V , \quad \eta_H = \eta_V - \epsilon_V .\tag{1.7}$$

The slow-roll parameters are thus related to the velocity and acceleration of the inflaton field along a field space trajectory. These are determined from an equation of motion

$$\ddot{\phi}_0 + 3H(t)\dot{\phi}_0 + V_\phi = 0 .\tag{1.8}$$

In the slow-roll limit, the potential is friction dominated, therefore the kinetic energy  $\dot{\phi}_0$  is very small compared to the other two terms. Together with the assumption that  $|\dot{H}|/H \ll 1$ , which follows from the assumption that  $\dot{\phi}$  is small and thus the motion of the inflaton is potential energy dominated, the above equation reduces to

$$3H(t)\dot{\phi}_0 = -V_\phi ,\tag{1.9}$$

which straightforwardly translates into the slow-roll parameters defined in (eq. 1.6) make sense. When  $\ddot{\phi}_0$  cannot be neglected, the Hamilton-Jacobi parameters (eq. 1.5) are still well defined. This is the so-called fast-roll limit, which cannot be used to

generate many  $e$ -folds of inflation. It can be used, however, to end inflation, by assuming that after some period of exponential expansion a change in the potential leads to a rapidly decreasing Hubble parameter. Then, via some reheating mechanism the kinetic energy of the inflaton must be transferred to other degrees of freedom to populate the universe with the particles we see today.

The above scenario assumes there is only one single light degree of freedom during inflation, that is not coupled to heavy degrees of freedom. Although a minimal assumption, it is certainly not a natural assumption. For example, it might be that there was more than one light field present during inflation, leading to models of multi-field inflation (eg. Groot Nibbelink and van Tent, 2000, 2002, Hwang and Noh, 2002, Wands et al., 2002, van Tent, 2004, Rigopoulos et al., 2006b, Seery and Lidsey, 2005, Rigopoulos et al., 2006a, 2007, Byrnes and Wands, 2006, Lalak et al., 2007b, Wands, 2008, Malik and Wands, 2009, Langlois et al., 2008c, Langlois and Renaux-Petel, 2008, Langlois et al., 2008a,b, Peterson and Tegmark, 2010). Another possibility is that the kinetic terms are noncanonical, with higher order derivative operators present (eg. Armendariz-Picon et al., 1999, Garriga and Mukhanov, 1999, Alishahiha et al., 2004, Bezrukov and Shaposhnikov, 2008, Barvinsky et al., 2008). Also, couplings to heavy degrees of freedom will lead to non-trivial results (chapters 4 and 5 and Tolley and Wyman, 2010, Chen and Wang, 2010b, Cremonini et al., 2010b). However, currently a large subset of the simplest models of single-field inflation remain perfectly compatible with current CMB precision measurements (Komatsu et al., 2010, Larson et al., 2010), predicting a nearly scale invariant power law inflation (Starobinsky, 1992, Adams et al., 2001, Tocchini-Valentini et al., 2005, Gong, 2005, Covi et al., 2006, Hunt and Sarkar, 2007, Ichiki et al., 2010, Peiris and Verde, 2010, Hamann et al., 2010)). Upcoming data, such as that from the Planck satellite promises to provide new handles on the overall shape of the spectrum and, particularly in combination with other data sets, could help us determine the precise nature of any possible features in it. If present, such features will lead to qualitative new tests on the single-field slow-roll paradigm (Kosowsky and Turner, 1995, Copeland et al., 1998) and could constitute strong evidence in favour of the existence of additional degrees of freedom present during the evolution of density perturbations as the universe inflated.

## 1.2.2 The power spectrum

Given an inflaton trajectory parametrised by a parameter  $\phi_0$ , one can define perturbations around the trajectory as

$$\phi = \phi_0 + \delta\phi . \tag{1.10}$$

## Chapter 1: Introduction

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This definition is not gauge invariant, as also the spacetime metric will experience perturbations. Therefore, it is convenient for the calculation of the perturbations to introduce a gauge-invariant parameter (Sasaki, 1986, Mukhanov, 1988)

$$Q \equiv \delta\phi + \frac{\dot{\phi}}{H}\psi, \quad (1.11)$$

with  $\psi$  the scalar perturbation associated with the metric. Then, for  $Q$  the equation of motion is

$$\frac{d^2 Q}{dt^2} + 3H \frac{dQ}{dt} - \frac{\nabla^2}{a^2} Q + m_{\text{eff}}^2 Q = 0, \quad (1.12)$$

with  $m_{\text{eff}}^2 = H^2(2 + 2\epsilon_H - 3\eta_H)$  for slow-roll inflation. Furthermore, a parameter  $v = aQ$  can be introduced, which allows a convenient expression of (eq. 1.12) in conformal time,  $dt = a d\tau$ ,

$$\frac{d^2 v}{d\tau^2} - \nabla^2 v + a^2 (H^2(2 - \epsilon) + m_{\text{eff}}) v = 0. \quad (1.13)$$

Considering the Fourier transformed equation,

$$\frac{d^2 v}{d\tau^2} - k^2 v + a^2 (H^2(2 - \epsilon) + m_{\text{eff}}) v = 0. \quad (1.14)$$

one sees there are two obvious limits. First, one has the short wavelength limit where  $k \gg aH$ . In this case (eq. 1.14) reduces to a normal oscillator equation. The other limit is when  $k^2 \ll aH$ , the late time limit, where the solution to (eq. 1.14) is  $v \rightarrow a \times \text{const.}$ . The behaviour changes when  $k \sim aH$ , when the wavelength  $k$  is of the same order as the Hubble radius, and thus crosses the horizon.

These perturbations are sourced by the everpresent quantum fluctuations at small scales, which are in a de Sitter universe described by a Bunch-Davies vacuum (Bunch and Davies, 1978). In order to match the classical perturbations of (eq. 1.13) these perturbations have to be quantised. One proceeds by Fourier transforming the perturbation equation, and expand the mode  $v(\mathbf{k}, \tau)$  as

$$(2\pi)^3 v(\mathbf{k}, \tau) = v(k, \tau) \hat{a}(\mathbf{k}) + v^*(k, \tau) \hat{a}^\dagger(\mathbf{k}), \quad (1.15)$$

with initial condition

$$v(k, \tau) = \frac{1}{\sqrt{2k}} e^{-ik\tau}, \quad (1.16)$$

since we need the growing mode. The mode that satisfies the initial condition is

$$v(k, \tau) = \frac{e^{-ik\tau}}{\sqrt{2k}} \frac{k\tau - i}{k\tau}, \quad (1.17)$$

which goes, well after horizon crossing, to

$$v(k, \tau) = \frac{i}{\sqrt{2k}} \frac{1}{k\tau} . \quad (1.18)$$

This derivation is for a single-field, the multi-field derivation is presented in section 4.3. The application to slow-roll two-field inflation is presented in section 5.4. In the remainder of this introductory section the single-field scenario will be discussed.

The initial curvature power spectrum is then given by the two-point correlation function of the curvature perturbations

$$P_R(\mathbf{k}) = \langle v(\mathbf{k}, \tau_{\text{end}})v(-\mathbf{k}, \tau_{\text{end}}) \rangle . \quad (1.19)$$

where statistical isotropy can be assumed to perform the sum over all angles  $P_R(k) = \sum_{\text{angles}} P_R(\mathbf{k})$ . For single-field slow-roll inflation one finds for the power spectrum

$$P_R(k, \tau_{\text{end}}) = \frac{H^2}{24\pi^2 M_{\text{pl}}^4 \epsilon} \Big|_{k=aH} \quad (1.20)$$

where the power spectrum is evaluated at horizon exit,  $k = aH$  and stays constant until  $\tau = \tau_{\text{end}}$ . For comparison with the cosmic microwave background, we need a spherical expansion (Lyth and Liddle, 2009)

$$v(\mathbf{k}) = \int_0^\infty dk \sum_{lm} v_{lm}(k, \tau_{\text{end}}) Z_{klm}(\mathbf{x}) \quad (1.21)$$

with  $v_{lm}(k, \tau_{\text{end}})$  the expansion coefficients of  $v(\mathbf{k}, \tau_{\text{end}})$  and

$$Z_{klm}(\mathbf{x}) \equiv \sqrt{\frac{2}{\pi}} k j_l(k|\mathbf{x}|) Y_{lm}(\theta, \phi) . \quad (1.22)$$

In this equation,  $j_l$  is the spherical Bessel function,  $(\theta, \phi)$  are the spherical directions of  $\mathbf{x}$  and  $Y_{lm}$  is the spherical harmonic. Then, integrating  $v_{lm}$  over the sphere and, using statistical isotropy, summing over  $l$  and  $m$ , one obtains

$$\langle v_{lm}^*(k, \tau_{\text{end}})v_{l'm'}(k', \tau_{\text{end}}) \rangle = (2\pi)^3 P_R(k) \delta(k - k') \delta_{ll'} \delta_{mm'} . \quad (1.23)$$

The scale dependence of  $P_R(k)$  is defined as

$$n - 1 \equiv \frac{d \log P_R(k)}{d \log k} . \quad (1.24)$$

Assuming a constant spectral tilt,  $n - 1$ , one finds that  $P_R(k) \propto k^{n-1}$ . From (eq. 1.20) we obtain, using  $d \log(aH) \simeq Hdt$  and

$$-\frac{d(\log H)}{DN} \simeq -\epsilon, \quad -\frac{d(\log \epsilon)}{DN} \simeq 4\epsilon - 2\eta, \quad (1.25)$$

where  $dN = -Hdt$ , that the spectral tilt is related to the slow-roll parameters as

$$n_s = 1 - 6\epsilon + 2\eta. \quad (1.26)$$

For multi-field inflation, also entropy perturbations, commonly known as isocurvature perturbations, are possible. These perturbations do not change the energy density but redistribute energy differently among particle species. They occur when perturbations in a direction normal to the inflaton trajectory are generated. Entropy perturbations are defined similarly as curvature perturbations, as a variance of a statistical field. Given an isocurvature perturbation  $S(\mathbf{x}, \tau) = (\delta n_i(\mathbf{x}, \tau))/n_i$ , evaluated at a slice of uniform energy density, we define the isocurvature power spectrum

$$P_S = \langle S_{\mathbf{x}} S_{-\mathbf{x}} \rangle \quad (1.27)$$

and similarly a cross correlation power spectrum

$$P_{RS} = \langle v_{\mathbf{x}} S_{-\mathbf{x}} \rangle. \quad (1.28)$$

Besides curvature and isocurvature perturbations, also tensor perturbations are possible. Sourced by perturbations of the metric, they have a different energy dependence

$$P_h(k) = \frac{2H^2}{3\pi^2 M_{\text{Pl}}^4} \Big|_{k=aH}, \quad (1.29)$$

so that the tensor to scalar ratio  $r$  becomes (Liddle and Lyth, 1992)

$$r \equiv \frac{P_h(k)}{P_R(k)} = 16\epsilon. \quad (1.30)$$

This ratio can be used to determine the necessary field variation

$$r < 0.003 \left( \frac{50}{N} \right)^2 \left( \frac{\Delta\phi}{M_{\text{Pl}}} \right)^2, \quad (1.31)$$

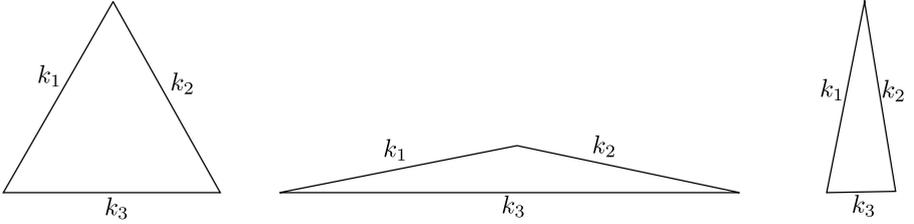
which is super-Planckian for large field inflationary models (Lyth, 1997).

### 1.2.3 Nongaussianities

One of the prime objectives of the Planck mission is to put better constraints on the existence of nongaussianities (Planck collaboration, 2006). Nongaussianities are given by the three point function (see Bartolo et al., 2004, for a review)

$$\langle v(\mathbf{k}_1, \tau_{\text{end}})v(\mathbf{k}_2, \tau_{\text{end}})v(\mathbf{k}_3, \tau_{\text{end}}) \rangle = \left(\frac{3}{5}\right)^3 (2\pi)^3 \delta(\mathbf{k}_1 + \mathbf{k}_2 + \mathbf{k}_3) F(k_1, k_2, k_3). \quad (1.32)$$

In this equation, the  $\delta$ -function ensures momentum conservation, meaning that the vectors  $k_1$ ,  $k_2$  and  $k_3$  should form a triangle in phase space. The shape of the triangle is determined by the shape function  $F(k_1, k_2, k_3) = f_{NL}S(k_1, k_2, k_3)$ , which consists of an identity shape function  $S(k_1, k_2, k_3)$  and a factor  $f_{NL}$  that determines the relative weight of this shape.



**Figure 1.4:** A pictorial version of three commonly used shape functions for the nongaussianities

Commonly, three momentum triangle shapes are studied (see figure 1.4). First, the local shape corresponds to  $k_1 \approx k_2$ , while  $k_3 \rightarrow 0$ . Second, the equilateral shape corresponds to  $k_1 \approx k_2 \approx k_3$ . Finally, the orthogonal shape is defined by the momenta  $k_1 \approx k_2$ , while  $k_3 \rightarrow \infty$ . These relations are used to define identity shape functions, that are multiplied with a factor  $f_{NL}$  as a measure of the amount of nongaussianities in the cosmic microwave background. Current best estimates, at 68% confidence level, are  $f_{NL}^{(local)} = 32 \pm 21$ ,  $f_{NL}^{(eq.)} = 26 \pm 140$  and  $f_{NL}^{(ort.)} = 202 \pm 104$  (Komatsu et al., 2010).

For a free field theory  $f_{NL} = 0$ . For single-field inflation, the only source for nongaussianities is due to the nonlinear character of gravity, which generates local nongaussianities  $f_{NL}^{(local)}(5/12)(1 - n_s) \approx 10^{-2}$  (Maldacena, 2003, Acquaviva et al., 2003)

## 1.3 Heavy physics and inflation

Inflationary physics takes place at the highest energy scales — the natural scale for inflation is  $O(10^{14})\text{GeV}$  — where string theory becomes relevant. It is for this reason that many attempts have been made to find string theory models for inflation. A short overview is given in the next subsection. In this subsection, also the vacuum structure of string theory is discussed. Almost all 4 dimensional reductions of string theory bring many degrees of freedom, moduli, that need some mechanism to be stabilised. No matter the mechanism, these modes will be lighter than the Planck mass and thus be relatively light as seen from the point of view of inflation. This warrants a study of an effective field theory description of string inflation, which in turn requires a discussion of the role, or rather the removal, of these light fields. A short overview of this method is presented in the second subsection, a further study is the topic of this thesis.

### 1.3.1 String theory and inflation

#### Flux compactifications

String theory is conventionally and conveniently defined in ten dimensions. In order to get an effective four dimensional theory, six dimensions have to be compactified on a six dimensional manifold. Using the supergravity descriptions of string theory, it is known that compactifying on a torus leaves the maximal amount of supersymmetry, while compactifying on manifolds with less internal symmetry leads to less supersymmetric effective theories (Aspinwall, 2000, Freedman and van Proeyen, 2009). Compactifying IIB supergravity on a Calabi-Yau manifold leads to an  $\mathcal{N} = 2$  supergravity. Adding orientifolds, branes and/or fluxes can reduce this to  $\mathcal{N} = 1$  supergravity. For this reason, Calabi-Yau manifolds are widely used as compact manifold for compactification.

Calabi-Yau manifolds have many internal symmetries (Candelas and de la Ossa, 1991) that will show up as light degrees of freedom, moduli, in the effective description, unless some stabilising potential is generated. Work by Gukov et al. (2000), Dasgupta et al. (1999), Greene et al. (2000) and Giddings et al. (2002) has shown that three form fluxes can provide the necessary potential for the complex structure moduli and the dilaton, leaving only the volume moduli unfixed. A first method to fully stabilise all moduli, the so called KKLT framework, was provided by Kachru et al. (2003a), where nonperturbative effects provide a potential for the volume moduli. In the KKLT paper, also a method to provide an additional contribution to the vacuum

expectation value is provided. This is necessary, as without it the model predicts a vacuum in anti-de Sitter space, while our universe has always been characterised by a positive vacuum expectation value.

Another method of stabilising all moduli was found by Balasubramanian et al. (2005), the so-called large volume compactifications. The method invoked is that a combination of stringy corrections ( $\alpha'$  corrections) and nonperturbative effects generate a potential that is zero at the origin and at infinity, while it is negative in between. It can be shown that at an exponentially large volume there exists a minimum with a negative vacuum expectation value and broken supersymmetry. These models generally predict a low scale of supersymmetry breaking and can be phenomenologically successful (eg. Conlon et al., 2005, 2007).

### Inflation in string theory

A good overview of the current state-of-the-art of string theory inflation is given in McAllister and Silverstein (2008), Baumann and McAllister (2009) and references therein. String theory models fall in two broad classes, small-field and large-field models, depending on the range of variation of the inflaton. Small-field models are models where the inflaton degree of freedom is represented by a parameter that can only move small distances on the internal compactification manifold, and by invoking the Lyth bound (eq. 1.31) thus operates at low energy scales. The typical model consists of a  $D$ -brane moving along a warped throat (Klebanov and Strassler, 2000). A first model, using the potential of a  $D3-\overline{D3}$  brane pair in a KKLT compactification, was provided by Kachru et al. (2003b). Further work has shown that the true story is more complicated (eg. Baumann et al., 2008, 2007, Krause and Pajer, 2008) although the strongly coupled dynamics of branes can also be used for inflation (Silverstein and Tong, 2004, Alishahiha et al., 2004). An overview of other available degrees of freedom can be found in Binetruy and Gaillard (1986)

In general, the KKLT framework has led to a large class of inflation models. Further addition of an extra nonperturbative term leads to the racetrack scenario (Blanco-Pillado et al., 2004, Lalak et al., 2007c, Blanco-Pillado et al., 2006b), Kähler moduli inflation (Conlon and Quevedo, 2006, Blanco-Pillado et al., 2010) or other volume moduli inflation models (Misra and Shukla, 2008, Conlon et al., 2008, Badziak and Olechowski, 2009, 2008). Models that use the volume modulus as inflaton also draw inspiration from the large volume compactification framework.

As discussed above, inflation using a string degree of freedom allows the inflaton to move only sub-Planckian distances in field space and thus allows only for a low scale of inflation (Kallosh and Linde, 2007b). A detection of tensor modes (eq. 1.30),

pointing to a large scale of inflation (eq. 1.31) thus suggests a non-stringy inflation mechanism. A loophole in this argument is exploited by monodromy inflation (Silverstein and Westphal, 2008, McAllister et al., 2010), by  $D$ -branes and axions respectively. Later models include and Cicoli et al. (2009), Kaloper and Sorbo (2009) and Dong et al. (2010). Furthermore, renewed interest in shift symmetries (Kallosh and Linde, 2010, eq.) also suggests the possibility of large field models.

The complicated vacuum landscape has sparked many multi-field modes of stringy inflation (eg. Groot Nibbelink and van Tent, 2000, 2002, Hwang and Noh, 2002, Wands et al., 2002, van Tent, 2004, Rigopoulos et al., 2006b, Seery and Lidsey, 2005, Rigopoulos et al., 2006a, 2007, Byrnes and Wands, 2006, Lalak et al., 2007b, Wands, 2008, Malik and Wands, 2009, Langlois et al., 2008c, Langlois and Renaux-Petel, 2008, Langlois et al., 2008a,b, Peterson and Tegmark, 2010). Given the presence of multiple fields and interactions, one expects observable features in the power spectrum (chapter 5, Cremonini et al., 2010a,b) and nongaussianities, see chapter 5. There is a lot of work on this subject, see eg. Maldacena (2003), Bernardeau and Uzan (2002), Creminelli (2003), Bartolo et al. (2004), Rigopoulos et al. (2006a), Seery and Lidsey (2005), Langlois et al. (2008a), Langlois et al. (2008b), Langlois et al. (2008c), Tolley and Wyman (2010), Chen and Wang (2010b) and Barnaby (2010).

An important feature is that almost all models are set up such that the inflationary physics is happening in a sector well separated from all other physics and that the sectors are only coupled due to gravity. In principle, one could then integrate out these hidden degrees of freedom. However, integrating out physics is very challenging, therefore this separation is used as a justification to truncate the additional degrees of freedom. In this thesis, however, it is shown that such truncations are usually not well justified, and discusses the non-decoupling of these gravitationally coupled degrees of freedom. Truncation only makes sense when it is done consistently, such that the equation of motion obtained from the truncated theory is the same as the equation of motion obtained from the full theory. In this thesis, it is shown that consistently truncating degrees of freedom is far more subtle than usually assumed. A recent application of this knowledge in terms of the  $\eta$ -problem, the problem that the natural scale for  $\eta$  is  $O(1)$  instead of  $O(10^{-3})$  is given by Hardeman et al. (2010).

### 1.3.2 Supergravity and effective field theory

In this thesis, we will study the applicability of supergravity as an effective theory. For this to be possible, it is necessary to consistently truncate heavy degrees of freedom, as discussed in chapter 2. It is generally assumed that having gravitational strength couplings between two sectors is enough to truncate one of these sectors. Before dis-

cussing and properly defining this statement later in this section, let us first introduce the notation used.

### Notation and conventions

Supergravity is defined by an action

$$S = M_{\text{Pl}}^2 \int d^4x \sqrt{g} \left[ \frac{1}{2} R - g^{\mu\nu} G_{I\bar{J}} \nabla_\mu \xi^I \nabla_\nu \bar{\xi}^{\bar{J}} - V M_{\text{Pl}}^2 \right], \quad (1.33)$$

in which  $G^{I\bar{J}}$  is the inverse field space metric  $G_{I\bar{J}} = \partial_I \partial_{\bar{J}} G$  and  $g_{\mu\nu}$  is the spacetime metric with associated Riemann scalar  $R$ . Greek indices run over spacetime coordinates  $\{\mu, \nu\}$ , capital indices run over all fields  $\{I, \bar{J}\}$ . For calculational convenience we have defined the scalar fields  $\xi$  and functions  $V$ ,  $K$  and  $W$  to be dimensionless. The ( $F$ -term) potential  $V$  of the scalar sector is defined as

$$V = e^G (G_I G^I - 3). \quad (1.34)$$

Through the metric defined as above and  $G_I = \partial_I G$  the action (eq. 1.33) is completely specified by the real Kähler function  $G(\xi, \bar{\xi})$ , which is, when  $W \neq 0$ , related to global supersymmetry quantities through

$$G(\xi, \bar{\xi}) = K(\xi, \bar{\xi}) + \log(W(\xi)) + \log(\bar{W}(\bar{\xi})) \quad (1.35)$$

in terms of the real Kähler potential  $K(\xi, \bar{\xi})$  and the holomorphic (dimensionless) superpotential  $W(\xi)$ . The definition for  $G$  is convenient as it is invariant under Kähler transformations, i.e. it is invariant under the simultaneous transformation of

$$\begin{aligned} K(\xi, \bar{\xi}) &\rightarrow K(\xi, \bar{\xi}) + f(\xi) + \bar{f}(\bar{\xi}) \quad \text{and} \\ W(\xi) &\rightarrow e^{-f(\xi)} W(\xi), \end{aligned} \quad (1.36)$$

for an arbitrary holomorphic function  $f(\xi)$ . Furthermore, the first and second derivatives of the potential (eq. 1.34) are easily calculated and read

$$\nabla_I V = e^G (G_I + G^{\bar{J}} \nabla_I G_{\bar{J}}) + V G_I, \quad (1.37)$$

$$\begin{aligned} \nabla_I \nabla_J V &= e^G (G_{I\bar{J}} + \nabla_J G_K \nabla_I G^{\bar{K}} - \mathcal{R}_{I\bar{J}K\bar{L}} G^{\bar{K}} G^{\bar{L}}) + G_I V_J + G_J V_I \\ &\quad + (G_{I\bar{J}} - G_I G_{\bar{J}}) V, \end{aligned} \quad (1.38)$$

$$\nabla_I \nabla_J V = e^G (2\nabla_I G_J + G^K \nabla_I \nabla_J G_K) + G_I V_J + G_J V_I + (\nabla_I G_J - G_I G_J) V. \quad (1.39)$$

In the action (eq. 1.33) we have denoted the gauge covariant derivatives by  $\nabla_\mu \xi^I = \partial_\mu \xi^I - A_\mu^a k_a^I(\xi)$ , and  $k_a^I(\xi)$  are the Killing vectors that define the gauge transformations of the scalars,

$$\delta_{\text{gauge}} \xi^I = k_a^I(\xi) \alpha^a, \quad a = 1, \dots, n_v, \quad (1.40)$$

where  $\alpha^a$  are the gauge parameters. The kinetic terms of the gauge fields are determined by the (holomorphic) gauge kinetic functions  $f_{ab}(\xi)$

$$\mathcal{L}_{\text{gauge}} = -\frac{1}{4} (\text{Re} f_{ab}) F_{\mu\nu}^a F^{b\mu\nu} + \frac{1}{4\sqrt{-g}} (\text{Im} f_{ab}) F_{\mu\nu}^a \epsilon^{\mu\nu\rho\sigma} F_{\rho\sigma}^b. \quad (1.41)$$

The scalar potential includes a contribution from  $F$ -terms and  $D$ -terms

$$V = V_F + V_D, \quad (1.42)$$

where  $V_F$  and  $V_D$  can be written in as a function of the auxiliary fields of the chiral and gauge superfields,  $F^I$  and  $D^a$  respectively,

$$V_F = G_{IJ} F^I F^{\bar{J}} - 3e^G = e^G (G^{I\bar{J}} G_I G_{\bar{J}} - 3), \quad (1.43)$$

$$V_D = \frac{1}{2} \text{Re}(f_{ab}) D^a D^b. \quad (1.44)$$

The auxiliary fields have equations of motion that can be solved algebraically in terms of the chiral fields and read

$$F^I = e^{G/2} G^{I\bar{J}} G_{\bar{J}}, \quad (1.45)$$

$$D^a = i(\text{Re} f)^{-1ab} k_b^I G_I = -i(\text{Re} f)^{-1ab} k_b^{\bar{I}} G_{\bar{I}}. \quad (1.46)$$

The two expressions given for the  $D$ -terms are equivalent due to the gauge invariance of the Kähler function  $G(\xi, \bar{\xi})$  (Groot Nibbelink and van Holten, 2000, Binetruy et al., 2004)

$$\delta_{\text{gauge}} G = (k_a^I G_I + k_a^{\bar{I}} G_{\bar{I}}) \alpha^a = 0, \quad \text{for all } a = 1, \dots, n_v. \quad (1.47)$$

In this thesis we will assume that there are no constant Fayet-Iliopoulos terms (Fayet and Iliopoulos, 1974) present. Fayet-Iliopoulos terms require a more careful treatment that is outside the scope of this thesis.

The  $\mathcal{N} = 1$  supersymmetry transformations of the fermions in the chiral and vector multiplets  $\chi^I$  and  $\lambda^a$  are

$$\begin{aligned} \delta \chi^I_L &= \frac{1}{2} \gamma^\mu \nabla_\mu \xi^I \epsilon_R - \frac{1}{2} e^{\frac{1}{2}K} K^{I\bar{J}} \mathcal{D}_{\bar{J}} \bar{W} \epsilon_L, \\ \delta \lambda^a &= \frac{1}{4} \gamma^{\mu\nu} F_{\mu\nu}^a \epsilon + \frac{1}{2} i D^a \gamma_5 \epsilon. \end{aligned} \quad (1.48)$$

Here  $\epsilon$  is the parameter of the supersymmetry transformations, and  $\gamma^\mu$  represent the gamma matrices as usual. The subscripts  $R$  and  $L$  of the fermions stand for right and left chirality respectively,

$$\chi^I_R = \frac{1}{2}(1 - \gamma^5)\chi^I_R \quad \chi^I_L = \frac{1}{2}(1 + \gamma^5)\chi^I_L \quad (1.49)$$

From (eq. 1.48) we can see that in a homogeneous background ( $\nabla^\mu \xi^I = F^a_{\mu\nu} = 0$ ), a set of necessary conditions for unbroken supersymmetry is

$$\mathcal{D}_I W = 0 \quad \text{for all} \quad I = 1, \dots, n_c. \quad (1.50)$$

Equivalently this condition can be written in terms of the Kähler function as

$$\partial_I G(\xi, \bar{\xi}) = 0 \quad \text{for all} \quad I = 1, \dots, n_c, \quad (1.51)$$

which, using (eq. 1.37) immediately shows that supersymmetric solutions are automatically extrema of the scalar potential (eq. 1.34). Furthermore, note that although it is always possible to break supersymmetry spontaneously by non-vanishing  $F$ -terms and zero  $D$ -terms (eq. 1.48), the relations (eq. 1.45) and (eq. 1.46) imply that non-vanishing  $D$ -terms necessarily require non-vanishing  $F$ -terms, and therefore supersymmetry can never be broken by  $D$ -terms alone (Choi et al., 2005).

The result (eq. 1.51) implies, together with the expression for the scalar potential (eq. 1.43) and (eq. 1.44), that supersymmetric critical points  $\xi_0^I$  with non vanishing superpotential  $W(\xi_0) \neq 0$  always have a negative vacuum energy, i.e. they are Anti-de Sitter critical points

$$V(\xi_0) = -3e^{G(\xi_0)} < 0. \quad (1.52)$$

Interestingly, supersymmetric critical points are always perturbatively stable, regardless of being local minima, maxima or saddle points. The reason is that in an Anti-de Sitter background a fluctuation with a tachyonic mass might still be stable as long as it satisfies the Breitenlohner-Freedman bound (Breitenlohner and Freedman, 1982)

$$m^2 \geq \frac{3}{4} V(\xi_0), \quad (1.53)$$

which is always fulfilled by supersymmetric critical points.

### Gravitational couplings in supergravity and rigid supersymmetry

Having introduced the notation, we can now focus on defining gravitational couplings as discussed in this thesis. As is clear from (eq. 1.46)  $D$ -terms can never appear

without  $F$ -terms. Therefore, in the following discussion we focus in the simpler case of only  $F$ -terms. The only effect  $D$ -terms can have is coupling decoupled sectors via gauge couplings, leading to an entirely different scenario that is not discussed in this thesis.

To describe a two-sector system we will consider a minimally coupled scenario (Cremmer et al., 1983a, Binetruy and Gaillard, 1985)

$$G(L, \bar{L}, H, \bar{H}) = G^{(1)}(L, \bar{L}) + G^{(2)}(H, \bar{H}) , \quad (1.54)$$

with  $L, H$  denoting the fields in the two sectors respectively. In the following, we will take the indices  $\{i, \bar{j}\}$  to run over the  $L$  fields, while  $\{\alpha, \bar{\beta}\}$  denote the fields in the  $H$  sector. The  $L$  fields are assumed to be in the visible sector and thus allowed to be dynamical, while the  $H$  fields reside in another sector which is assumed not to take part in the dynamics and is hence called hidden sector. In chapter 2, a mass hierarchy between a light visible sector  $L$  and a heavy hidden sector  $H$  is present, which is why the sectors are labelled with  $L$  and  $H$ . However, currently  $L$  and  $H$  can be of arbitrary mass and are thus not necessarily light or heavy. This split of the Kähler function  $G(L, \bar{L}, H, \bar{H})$ , (eq. 1.54), is invariant under Kähler transformations in each sector separately (chapter 2 and Choi et al., 2004, de Alwis, 2005a,b, Achúcarro and Sousa, 2008) and thus defines a sensible way of splitting up the action in multiple sectors. In terms of  $K$  and  $W$ , this definition has a conventional separation of the Kähler function

$$K(L, \bar{L}, H, \bar{H}) + \log |W(L, H)|^2 = K^{(1)}(L, \bar{L}) + K^{(2)}(H, \bar{H}) + \log |W^{(1)}(L)W^{(2)}(H)|^2 , \quad (1.55)$$

but the superpotentials in each sector combine multiplicatively rather than add.

Let us illustrate the importance of this multiplicative superpotential in the situation in which the hidden sector resides in a supersymmetric vacuum, i.e.  $\partial_a V(H_0) = 0$  and  $\partial_a G^{(2)}(H_0) = 0$ . We write the superpotential of the hidden sector as  $W^{(2)}(H) = W_0^{(2)} + W_{\text{global}}^{(2)}(H - H_0)$ . The second term in this expression is what determines the potential for fluctuations around the minimum of the hidden sector, while the first constant term is just an overall contribution and hence not interesting for the internal hidden sector dynamics at energies much less than the Planck scale. However, for the gravitational dynamics and the remaining  $H^\alpha$  sector this ‘‘vacuum energy contribution’’  $W_0^{(2)}$  is of crucial importance as it sets the scale of the potential (Davis and Postma, 2008, Hardeman et al., 2010)

$$V = e^{K^{(2)}} |W_0^{(2)}|^2 e^{G^{(1)}} \left( G_i^{(1)} G^{(1)i} - 3 \right) , \quad (1.56)$$

which is evaluated at  $H = H_0$  such that all terms depending on  $W_{\text{global}}^{(2)}$  vanish. The normal practise of setting  $W_0^{(2)}$  to zero as an overall contribution to the hidden sector is neglecting the fact that gravity also feels the constant part of the potential energy, as opposed to field theory. The inflationary sector feels the presence of the hidden sector through this coupling and as such it may be more intuitive to regard  $W_0^{(2)}$  to contain information about the inflationary sector rather than the hidden sector. Making a similar split in  $W^{(1)}$ , the constant part  $W_0^{(1)}$  is the overall contribution to the hidden sector due to the inflaton sector.

The multiplicative superpotential also means that the zero-gravity limit to a global supersymmetry is more subtle than just taking  $M_{\text{Pl}} \rightarrow \infty$  as is usually done. One must first determine a ground state which sets  $W_0^{(1)}$  and  $W_0^{(2)}$ , and then send both  $W_0^{(1)} \rightarrow \infty$  and  $W_0^{(2)} \rightarrow \infty$  in such a way that the combinations  $W_0^{(1)} W_{\text{global}}^{(2)}$  and  $W_0^{(2)} W_{\text{global}}^{(1)}$  remain constant. The total superpotential

$$W = W_0^{(1)} W_0^{(2)} + W_0^{(1)} W_{\text{global}}^{(2)} + W_0^{(2)} W_{\text{global}}^{(1)} + W_{\text{global}}^{(1)} W_{\text{global}}^{(2)} \quad (1.57)$$

then consists of an overall infinite contribution, a finite sum of two terms and a negligible product. Only in this decoupling limit, does one recover the two independent global supersymmetry sectors with the naive additive behaviour in both the superpotential and the Kähler potential:

$$\begin{aligned} K(L, \bar{L}, H, \bar{H}) &= K^{(1)}(L, \bar{L}) + K^{(2)}(H, \bar{H}) , \\ W(L, H) &= W^{(1)}(L) + W^{(2)}(H) . \end{aligned} \quad (1.58)$$

However, one cannot use this split (eq. 1.58) and couple gravity back in (Davis and Postma, 2008). As explained, in supergravity the definition (eq. 1.58) is not invariant under Kähler transformations in each sector separately and is valid only in a specific Kähler frame or, say, gauge dependent (Achúcarro and Sousa, 2008). Another way to understand the result is to realise that the definition (eq. 1.58) does not lead to a Kähler metric and mass matrix that can be made block diagonal in the same basis (chapter 2), and thus there is no sense of “independent” sectors.

Insisting on the separate Kähler invariance of (eq. 1.54), the two-sector action (eq. 1.33) reads

$$S = M_{\text{Pl}}^2 \int d^4x \sqrt{g} \left[ \frac{1}{2} R - g^{\mu\nu} (G_{ij}^{(1)} \partial_\mu L^i \partial_\nu \bar{L}^j + G_{\alpha\beta}^{(2)} \partial_\mu H^\alpha \partial_\nu \bar{H}^\beta) - V M_{\text{Pl}}^2 \right], \quad (1.59)$$

with

$$V(L, \bar{L}, H, \bar{H}) = e^{G^{(1)} + G^{(2)}} (G_i^{(1)} G^{(1)i} + G_\alpha^{(2)} G^{(2)\alpha} - 3) . \quad (1.60)$$

We will allow ourselves to drop the sector label from  $G$  in the remainder, since  $G_L^{(1)} = G_L$  and similarly for  $H$ .

### Non-decoupling in effective theories

After defining gravitational strength couplings we can continue the discussion on integrating out degrees of freedom in general field theories. As discussed in chapter 2, the requirement for truncating a degree of freedom is

$$\left. \frac{\delta \widehat{S}}{\delta L} \right|_{H_0} = \frac{\delta \widehat{S}|_{H_0}}{\delta L} = \frac{\delta S}{\delta L}, \quad (1.61)$$

where  $L$  and  $H$  point to a visible and hidden sector. Furthermore,  $\widehat{S}$  refers to the full action for both the  $L$  and  $H$  sector, while  $S$  is the effective action for the  $L$  sector only.

Due to gravity, in any effective theory the gravitational force will couple everything to everything. This leads to Planck-suppressed corrections that are usually not relevant due to the huge scale difference between everyday physics and the Planck mass. Yet, in case of inflation there is a difference. On the one hand, the scale of the problem is much closer to the Planck scale. On the other hand, fields will move considerable distances in field space, also probing Planckian corrections. As shown in chapters 4 and 5 gravitational size couplings in non-linear sigma models can lead to corrections on the light degree of freedom. These corrections will manifest themselves as a reduced speed of sound for the light perturbations, leading to features in the power spectrum and nongaussianities.

In the context of supergravity, chapter 3 focuses on a model with a Kähler function of the form (eq. 1.54), where one of the sectors is in its supersymmetric minimum. This meets the requirement for consistent decoupling as derived in chapter 2. In fact, due to the separable Kähler function the field space manifold is actually a product manifold, making it possible to consistently decouple the heavy sector globally, thus consistently truncating the heavy physics (Groot Nibbelink and van Holten, 2000). However, instead of focusing on the supersymmetry broken sector, we focus on the supersymmetric sector, that *will* receive corrections from the supersymmetry broken sector. The reason is that the supersymmetry broken sector can provide an uplifting term, allowing de Sitter models in supergravity. Yet, we show that the coupling of the supersymmetry broken sector destabilises uplifted local minima after some amount of uplifting. In contrast, local maxima, that are actually stable in anti-de Sitter space due to the Breitenlohner-Freedman bound (Breitenlohner and Freedman, 1982), become stable when uplifted to Minkowski or de Sitter space.

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## CHAPTER 2

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# Consistent decoupling of heavy scalars and $\mathcal{N} = 1$ supergravity

The viability of theories based on extra dimensions, in particular string theory, relies on being able to stabilise and integrate out the fields (moduli) that describe the shapes and sizes of those extra dimensions, for which so far there is no observational evidence. In flux compactifications (Giddings et al., 2002) some moduli are stabilised at a high energy scale and decouple from the low energy theory. From that moment on we never see them in the effective low energy description.

Unlike in global supersymmetry, complete decoupling is of course impossible in supergravity – even in principle – because gravity couples to all fields. At low energies one is usually satisfied with gravitational strength couplings between the heavy, stabilised, fields and the low energy fields. However, such interaction terms are of order  $O(G_{\text{Newton}}E^2) = O(E^2/M_P^2)$ , where  $E$  is the energy scale and  $M_P \approx 2.4 \times 10^{18}\text{GeV}$  the reduced Planck mass. Even if they are strongly suppressed at low energy and in particle accelerators, these couplings become sizable at the energy scales relevant to the early Universe, and one must look for a more robust definition of decoupling that can be extrapolated over a wide range of energy scales. The purpose of this chapter is to provide such a definition, and a simple test of whether it holds in specific models.

There are at least two situations in which the details of decoupling are important. One is supersymmetry breaking, which will affect the heavy fields in a way that is not accounted for in the low energy effective action. Uplifting in KKLТ scenarios (Kachru et al., 2003a) is a prime example. The second is inflation with moduli stabilisation, because the inflaton, which is a low energy field in this language, can have its expectation value vary over many Planck-masses.

Here we take a bottom-up approach and try to find for what types of Supergravity couplings we can be sure that the heavy moduli will not shift from their expectation values due to low energy processes. We do not require small gravitational coupling to the light(er) fields because instead we rely on supersymmetry to partially protect the expectation values of the heavy moduli.

It must be stressed that what we are proposing here, building on arguments by Choi et al. (2005), de Alwis (2005b), de Alwis (2005a), Binetruy et al. (2004) and Achúcarro and Sousa (2008), is a simple consistency test. It checks explicitly what is implicitly assumed by the very use of a low energy effective action. So it is somewhat surprising to find that the most common ansatz for decoupled fields in the literature, the standard “gravitational strength coupling” ansatz, generically fails the test. It partly explains the difficulties encountered in supergravity models of inflation with moduli stabilisation. The problem essentially disappears for consistently decoupled moduli (Davis and Postma, 2008, Achúcarro and Sousa, 2008).

## 2.1 Consistent decoupling of scalar fields in $\mathcal{N} = 1$ supergravity

In what follows we consider two sets of fields, heavy ( $H$ ) and light ( $L$ ), and assume the heavy fields are stabilised at an expectation value  $H = H_0$ , an extremum of the scalar potential for the heavy moduli. If the heavy field is a singlet under all low energy symmetries and its mass is large enough it will decouple from low energy phenomena and can be truncated, leaving an effective theory for the light degrees of freedom. To make this distinction, we will again use hatted quantities to indicate the full theory, including heavy and light fields, and unhatted quantities for the effective theory involving light fields only

$$S(L, \bar{L}) = \widehat{S}(H_0, \bar{H}_0, L, \bar{L}). \quad (2.1)$$

We are interested in the case in which the resulting effective theory is also described by  $\mathcal{N} = 1$  supergravity. In this case, there should be an effective  $K$  and  $W$  (or

$G$ ) depending only on the light fields, from which to compute the low energy action  $S$  and supersymmetry transformations

$$G[L, \bar{L}] = \widehat{G}[L, \bar{L}, H_0, \bar{H}_0], \quad (2.2)$$

$$\delta_\epsilon L = \hat{\delta}_\epsilon L|_{H_0} = f[L, G(L, \bar{L})], \quad (2.3)$$

$$\hat{\delta}_\epsilon H|_{H_0} = 0. \quad (2.4)$$

Notice that the  $F$ -terms (eq. 1.45) of the heavy fields must vanish because the supersymmetry transformations read,

$$\hat{\delta}_\epsilon H \sim \chi \epsilon, \quad \hat{\delta}_\epsilon \chi \sim \not{\partial} H \epsilon - \frac{1}{2} F \epsilon \quad (2.5)$$

and if the  $F$ -terms are non-zero a supersymmetry transformation will generate light fermions, related to the supersymmetry breaking in the heavy sector, that are not in the low energy effective action. Thus, the heavy fields cannot contribute to supersymmetry breaking, leading to

$$\partial_H \widehat{G}|_{H_0} = 0 \quad \text{or} \quad \widehat{\mathcal{D}}_H \widehat{W}|_{H_0} = 0, \quad (2.6)$$

(see also de Alwis, 2005b) where  $\widehat{\mathcal{D}}_i \widehat{W} = \partial_i \widehat{W} + (\partial_i \widehat{K}) \widehat{W}$  is the Kähler covariant derivative that transforms as  $\widehat{\mathcal{D}}_i \widehat{W} \rightarrow e^{-h(\epsilon)} \widehat{\mathcal{D}}_i \widehat{W}$  under Kähler transformations. Note that  $\widehat{\mathcal{D}}_H \widehat{W} = 0$  is the condition used in flux compactifications (Giddings et al., 2002) and by extension in KKLT (Kachru et al., 2003a) and LARGE volume scenarios (Balasubramanian et al., 2005), where the complex structure moduli are stabilised at a supersymmetric point before uplifting.

The Kähler metric should be block diagonal in the light and heavy fields when evaluated at  $H_0$ , otherwise propagators will mix these two sets of fields. Additionally, the truncation  $H = H_0$  must of course be a consistent truncation. This means that the equations of motion of the light fields derived from the effective theory are the same as the equations of motion obtained from the full theory. To zeroth order in the fluctuations of the heavy fields

$$\left. \frac{\delta \widehat{S}}{\delta L} \right|_{H_0} = \frac{\delta \widehat{S}|_{H_0}}{\delta L} = \frac{\delta S}{\delta L}, \quad (2.7)$$

ensuring that the fluctuations of  $H$  are not sourced by the light fields. In particular, the heavy fields should be singlets under the surviving gauge group at low energies, otherwise they remain coupled to the light fields by the gauge interaction. In what follows we will consider  $f_{ab}(L)$  independent of the heavy fields. In that case they do not contribute to the  $D$ -terms, which will only involve light fields.

## 2.2 Analysis of the consistency conditions

The heavy fields thus need to be stabilised at an expectation value  $H_0$ , where  $H_0$  is the solution to (eq. 2.6)

$$\left[ \partial_H \widehat{W}(H, L) + \partial_H \widehat{K}(H, \overline{H}, L, \overline{L}) \widehat{W}(H, L) \right] \Big|_{H_0} = 0 . \quad (2.8)$$

which implies  $\partial_H \widehat{V}|_{H_0} = 0$ . The LHS is some function of both the heavy and the light fields, let us call it  $\Phi(H, \overline{H}, L, \overline{L})$ . In general, the condition  $\Phi = 0$  (together with its complex conjugate  $\overline{\Phi} = 0$ ) relate the heavy and light fields. If we can solve for  $H$  we obtain an expression of  $H_0$  as a function of the light fields,

$$H = H_0(L, \overline{L}) , \quad (2.9)$$

which can be substituted back into  $\widehat{K}, \widehat{W}$  to give an effective action for the light fields

$$S(L, \overline{L}) = \widehat{S}(H_0(L, \overline{L}), \overline{H}_0(L, \overline{L}), L, \overline{L}) . \quad (2.10)$$

An immediate concern with the consistency of this procedure, pointed out in de Alwis (2005b), is that in general this leads to a non-holomorphic expression for the would-be effective superpotential  $W = \widehat{W}(H_0(L, \overline{L}), L)$ . However, this problem is easily avoided: it does not arise if  $\widehat{W}$  is independent of  $H$ . The case  $\widehat{W} = 0$  is obvious, so consider  $\widehat{W} \neq 0$ . It is always possible to perform a Kähler transformation that makes  $\widehat{W}$  constant

$$\widehat{W} \rightarrow 1 , \quad (2.11a)$$

$$\widehat{K} \rightarrow \widehat{K} + \log \widehat{W} + \log \overline{\widehat{W}} = \widehat{G} . \quad (2.11b)$$

In this so called Kähler gauge, (eq. 2.8) reads

$$\partial_H \widehat{G}(H, \overline{H}, L, \overline{L}) = 0 , \quad (2.12)$$

from which we can extract  $H = H_0(L, \overline{L})$  and make the previous substitution directly into the Kähler invariant function without any inconsistency (see also Curio and Spillner, 2007):

$$G = \widehat{G}(H_0(L, \overline{L}), \overline{H}_0(L, \overline{L}), L, \overline{L}) . \quad (2.13)$$

In fact, the issue is not whether  $H_0(L, \overline{L})$  is holomorphic but rather whether it is a (*non-trivial*) function at all. The assumption that the heavy fields are *stabilised* at

$H = H_0$  is simply the condition that  $H_0(L, \bar{L}) = \text{constant}$ . Any other dependence on the light moduli would translate into a constraint on the light fields which would have to be accounted for explicitly in the low energy action (de Alwis, 2005a). This is what we have to avoid.

To summarise, the (rather obvious) mathematical condition for the heavy fields to be truncated consistently with an expectation value  $H_0$  and to decouple from the low energy fields is that the system of equations

$$\partial_H \widehat{G} \equiv \Phi(H, \bar{H}, L, \bar{L}) = 0, \quad (2.14)$$

which is the same as (eq. 2.8) defined in the Kähler gauge (eq. 2.11), admits the constant solution

$$H = H_0(L, \bar{L}) = \text{const}, \quad \bar{H} = \bar{H}_0(L, \bar{L}) = \text{const}. \quad (2.15)$$

In spite of being obvious, this condition is not empty. For instance, we will see below that it *fails* generically for standard couplings of the form  $K = K_1 + K_2$  and  $W = W_1 + W_2$ . However, let us first consider two specific situations in which the decoupling condition does hold.

1. The consistency condition is trivially satisfied if the function  $\Phi(H, \bar{H}, L, \bar{L})$  has no explicit dependence on the light fields. In this case by integrating (eq. 2.14) one recovers the condition found in Binetruy et al. (2004)

$$\partial_H \widehat{G} = \Phi(H, \bar{H}) \rightarrow \widehat{G} = \widehat{G}_1(H, \bar{H}) + \widehat{G}_2(L, \bar{L}) \quad (2.16)$$

and it is obvious that the Kähler metric is block diagonal in this case. This ansatz has a long history which goes back to Cremmer et al. (1983a) and allows a detailed stability analysis of the heavy fields (Achúcarro and Sousa, 2008 and chapter 3), in particular in the context of  $F$ -term uplifting of flux compactifications.

2. On the other hand, this requirement is too restrictive. It is sufficient if the function  $\Phi(H, \bar{H}, L, \bar{L})$  factorises,

$$\Phi(H, \bar{H}, L, \bar{L}) = \Phi_1(H, \bar{H}, L, \bar{L}) \Phi_2(H, \bar{H}) = 0, \quad (2.17)$$

in which case we just solve  $\Phi_2 = \bar{\Phi}_2 = 0$  to get constant  $H_0, \bar{H}_0$ . We cannot give the general form of  $\widehat{G}$  for which this factorisation occurs, but it will certainly hold if  $\widehat{G}$  has the following functional form:

$$\widehat{G} = f(L, \bar{L}, g(H, \bar{H})), \quad (2.18)$$

since in that case (eq. 2.6) is replaced by

$$\partial_H g(H, \bar{H}) = 0 . \quad (2.19)$$

The first situation, (eq. 2.16), is a special case of (eq. 2.19), with  $\Phi_1$  constant. In both cases, the same condition that makes  $\widehat{G}_H|_{H_0} = 0$  also implies that the Kähler metric and the Hessian of  $V$  are block diagonal for any  $\Phi_1$ . Indeed, from (eq. 2.19) we find that

$$\widehat{G}_{LH}|_{H_0} = \partial_L \partial_{\bar{g}} f(L, \bar{L}, g(H, \bar{H})) \partial_H g(H, \bar{H})|_{H_0} = 0 \quad (2.20)$$

and further all mixed derivatives with only one derivative with respect to the heavy field vanish. As  $V_{LH}$  always contains terms  $\propto \widehat{G}_H$  or  $\propto (\partial_L)^n \widehat{G}_H$ , which vanish at  $H_0$ , the Hessian of  $V$  is block diagonal.<sup>1</sup>

## 2.3 Consistent decoupling compared to gravitational coupling in rigid supersymmetry

Finally, we stress that the condition derived here has no direct relation to the condition usually associated with gravitational strength coupling, see also the discussion in section 1.3.2. In fact, the ansatz

$$\widehat{K} = K_1(H, \bar{H}) + K_2(L, \bar{L}) , \quad (2.21a)$$

$$\widehat{W} = W_1(H) + W_2(L) \quad (2.21b)$$

does not satisfy the decoupling condition in general. Suppose (eq. 2.6) admits a constant solution  $H = H_0$ . Then

$$0 = \partial_H W_1|_{H_0} + \partial_H K_1|_{H_0} [W_1(H_0) + W_2(L)] , \quad (2.22)$$

which only holds if

$$\begin{aligned} \partial_H K_1|_{H_0} = 0 &\Rightarrow \partial_H W_1|_{H_0} = 0 \quad \text{or} \\ \partial_H K_1|_{H_0} \neq 0 &\Rightarrow W_2(L) = -\frac{\partial_H W_1|_{H_0}}{\partial_H K_1|_{H_0}} - W_1(H_0) \\ &= \text{const.} \end{aligned} \quad (2.23)$$

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<sup>1</sup>Note that it is always possible to block-diagonalise the Kähler metric or the Hessian of  $V$  at one point, but it is not necessarily the case that both block-diagonalisations are compatible, as we have here.

Another way to see this: since  $\widehat{\mathcal{D}}_H \widehat{W} = 0$  does not factorise, the (Kähler-gauge covariant) requirement that it is independent of the light fields is (see also Ben-Dayan et al., 2008)

$$\widehat{\mathcal{D}}_L (\widehat{\mathcal{D}}_H \widehat{W}) = 0. \quad (2.24)$$

Inserting the ansatz (eq. 2.21) then gives

$$\partial_H K_1|_{H_0} \partial_L W_2 = 0. \quad (2.25)$$

Unless  $K_1(H, \overline{H})$  has no linear terms or  $W_2(L) = \text{constant}$ , the condition will not be met. However, if  $W_2(L) = \text{constant}$  (e.g. no scale models, such as presented in Cremmer et al., 1983b, Giddings et al., 2002) then equation (eq. 2.16) holds and  $\widehat{W}$  is trivially a product. On the other hand, we can always expand  $K_1(H, \overline{H})$  around  $H_0$  and remove the linear terms by a Kähler transformation (eq. 1.36), but this spoils the separability of the superpotential (eq. 2.21b).

In other words, if two sets of fields are described by a separable Kähler function  $K = K_1(\text{heavy}) + K_2(\text{light})$ , the addition of their superpotentials does not respect the decoupling condition except in special cases (and, incidentally, neither does it guarantee gravitational strength couplings if  $K_1(\text{heavy}) = O(M_p^2)$ , as is usual for moduli).

## 2.4 Discussion

In this chapter we have studied how to truncate heavy scalars and moduli and their superpartners in  $\mathcal{N} = 1$  supergravity, subject to two *explicit* requirements. First, the expectation values of the heavy fields should be unaffected by low energy phenomena, in particular supersymmetry breaking. Second, the low energy effective action should be described by  $\mathcal{N} = 1$  supergravity. This is what we call *consistent decoupling*.

If the heavy fields are stabilised at a critical point of the potential, integration of the whole superfield requires that the  $F$ -terms should be zero (Binetruy et al., 2004). The criterion for consistent decoupling is that the expectation value of the heavy scalars  $H$  should not depend on the light fields  $L$  (de Alwis, 2005a). Our main result is a class of Kähler invariant functions that satisfy the condition, given in (eq. 2.18):

$$\widehat{G} = f(L, \overline{L}, g(H, \overline{H})).$$

This functional form guarantees that the Kähler metric and Hessian of  $V$  are simultaneously block diagonal in the heavy and light fields. It also allows the embedding of BPS solutions of the low energy effective theory into the full theory without

destroying their BPS character (if the  $F$ -terms of the heavy fields are zero and in the absence of constant Fayet-Iliopoulos terms, the supersymmetric transformation of the gravitino depends only on the light fields). We would expect the BPS character to survive quantum corrections – now in the full theory –. Thus, at least in this special case it would seem possible to “screen” the heavy, decoupled fields from the effects of (partial) supersymmetry breaking in the low energy sector.

We only have experimental access to  $G$ , the effective low energy theory, and there is a large class of supergravity models (read a landscape of compactifications), characterised by  $\widehat{G}$ , in which the low energy theory could be embedded. Here,  $\widehat{G}$  includes all stringy, perturbative and non-perturbative effects. The decoupling condition restricts the allowed functional form of  $\widehat{G}$  and therefore the class of models that are consistent with the assumption of decoupling that is implicit in our use of  $G$ . From the point of view of model building, it provides a simple test that has not been considered before. There are string compactifications which approximately satisfy the decoupling condition in the form (eq. 2.16), such as some LARGE volume scenarios (LVS) (Balasubramanian et al., 2005, Conlon et al., 2005, 2007, Conlon, 2008).

To see this, note first of all that the tree level or GKP limit (Giddings et al., 2002) of  $\widehat{G}$  satisfies (eq. 2.16) with the complex structure moduli and the dilaton  $S$  playing the role of the heavy fields. Assume the usual form for the leading non-perturbative and  $\alpha'$  corrections,  $\widehat{W} = W_{\text{GKP}}(H) + W_{\text{np}}(L)$ ,  $\delta\widehat{K} \sim 2(S + \bar{S})^{3/2}/\mathcal{V}$ , with  $\mathcal{V}$  is the volume modulus of the compact manifold. Ignoring for a moment the dilaton dependence of  $\delta\widehat{K}$ , we find for the complex structure moduli

$$\partial_H \widehat{G} = \partial_H K_{\text{heavy}}(H) + \frac{\partial_H W_{\text{GKP}}(H)}{W_{\text{GKP}}(H)} \left[ 1 + \delta(L, H) \right]^{-1}, \quad (2.26)$$

where  $\delta = W_{\text{np}}(L)/W_{\text{GKP}}(H)$ . Including dilaton effects adds a correction  $\delta \sim (S + \bar{S})^{3/2}/\text{vol}$  (whichever is larger). The condition of consistent decoupling is violated by the L-dependence of  $\delta$ .

A stabilised LVS or KKLТ vacuum, with nonperturbative corrections, is obtained from a Kähler potential and superpotential of the form

$$K = K_{c,s} - 2 \log \left[ e^{-\frac{3\phi_0}{2}} \mathcal{V} + \frac{\xi}{2} \left( \frac{-i(\tau - \bar{\tau})}{2} \right)^{3/2} \right], \quad (2.27)$$

$$W = W_0 + \sum_n A_n e^{i a_n \rho_n}. \quad (2.28)$$

Here,  $\tau$  represents the axio-dilaton field and  $\xi = -\zeta(3)\chi(M)/(16\pi^3)$ , where  $\chi(M)$  is the Euler characteristic. Furthermore,  $\rho_i$  are the complexified Kähler moduli which

in this specific case correspond to 4-cycles. Furthermore,  $a_i = 2\pi/K$ , with  $K \in \mathbb{Z}_+$ . From the Kähler potential  $K$  and superpotential  $W$  one obtains a scalar potential

$$\begin{aligned}
 V &= e^K \left[ G^{\rho_j \bar{\rho}_k} \left( a_j A_j a_k \bar{A}_k e^{i(a_j \rho_j - a_k \bar{\rho}_k)} + i \left( a_j A_j e^{i a_j \rho_j} \bar{W} \partial_{\bar{\rho}_k} K - a_k \bar{A}_k e^{-i a_k \bar{\rho}_k} W \partial_{\rho_j} K \right) \right) \right. \\
 &\quad \left. + 3\xi \frac{(\xi^2 + 7\xi\mathcal{V} + \mathcal{V}^2)}{(\mathcal{V} - \xi)(2\mathcal{V} + \xi)^2} |W|^2 \right] \\
 &\equiv V_{np1} + V_{np2} + V_{\alpha'} .
 \end{aligned} \tag{2.29}$$

It can be shown that for Calabi-Yau manifolds with a negative Euler characteristic the potential is positive at  $\mathcal{V} = 0$ , zero at  $\mathcal{V} \rightarrow \infty$  and negative for large  $\mathcal{V}$ . This construction can thus be used to generate a potential that allows for a vacuum solution at an exponentially large volume. As this solution is obtained after truncating the complex structure, it might be shifted as given by (eq. 2.26). In the case of an LVS vacuum with parameters  $A \sim 1$ ,  $W_{\text{GKP}}(H_0) \sim 10$ ,  $\mathcal{V} \sim 10^{10}$ ,  $Ae^{-a_4 \tau_4} \sim 1/\mathcal{V}$  (see Balasubramanian et al., 2005) it is negligible,  $\delta \sim O(10^{-10})$ , as the correction scales inversely with the volume.<sup>2</sup> In the mirror mediation scenarios (Conlon, 2008)  $\delta$  is even smaller.

Another well known solution to (eq. 2.29) is the KKLT solution (Kachru et al., 2003a). This solution is characterised by parameters  $A \sim O(1)$ ,  $W_{\text{GKP}}(H_0) \sim O(10^{-4})$ ,  $aL \sim O(10)$  and leads to a significant change of the vacuum obtained in the truncated theory as  $\delta \sim O(1)$ .

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<sup>2</sup>Berg et al. (2007) and Cicoli et al. (2008) suggest that string loop corrections to  $\hat{K}$  scale as  $(\mathcal{V})^{-2/3}$  and would lead to  $\delta < 10^{-6}$ . We thank M. Cicoli for this remark.



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## CHAPTER 3

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# **$F$ -term uplifting and the supersymmetric integration of heavy scalars**

### **3.1 Introduction**

The search for de Sitter (dS) vacua in string theory has received a lot of attention motivated by the need to construct realistic late-time cosmology scenarios. In 2003 Kachru et al. (KKLT) provided the first example of a mechanism to obtain stable de Sitter vacua in the framework of Type IIB string theory. Their two-step approach was to invoke background fluxes and non-perturbative effects in order to *freeze* the heavy moduli present in the compactification *while preserving supersymmetry*, and then add extra supersymmetry breaking effects in a controlled way, i.e. not interfering with moduli stabilisation, so that the anti-de Sitter (AdS) minimum would be uplifted to dS.

In practise, in the KKLT paper and in many sequels that discussed mechanisms of uplifting of the AdS minimum, it is assumed that the complex structure moduli are truncated before supersymmetry breaking effects are taken into account. The effective field theory left after freezing these fields is assumed to be  $\mathcal{N} = 1$  supergravity. In other words, the heavy moduli are truncated *supersymmetrically* (Binetrui et al., 2004) and are assumed to be consistently decoupled, as discussed in de Alwis

(2005b), de Alwis (2005a) and chapter 2, from the light fields.

The problem, as discussed in chapter 2, Choi et al. (2004), de Alwis (2005b), de Alwis (2005a), Achúcarro and Sousa (2008) and Ben-Dayan et al. (2008), is that in general it cannot be taken for granted that the supersymmetry breaking corrections added to the effective action are consistent with the process of *supersymmetrically* integrating out part of the moduli. The reason is very simple: the heavy fields should be integrated out in the *full theory*, including the supersymmetry breaking modifications. Any other way to proceed may lead to inconsistencies. For example, the minimum where the moduli were stabilised might shift by the supersymmetry breaking effects, in which case the low energy effective theory would have the heavy fields – the complex structure moduli in this case – fixed at a point which is not even an extremum of the scalar potential.

In this chapter we will study a mechanism of  $F$ -term uplifting (Gomez-Reino and Scrucca, 2006b,a, 2007) consistent with the supersymmetric integration of the heavy moduli (chapter 2 and Achúcarro and Sousa, 2008). The basic idea of  $F$ -term uplifting consists of adding an extra sector to the theory defined by the Kähler and complex structure moduli which breaks supersymmetry separately, lifting the vacuum to dS. In order to avoid that the interactions between the two sectors spoil the stabilisation of moduli it is required that they are only weakly coupled. In the original papers of  $F$ -term uplifting this was achieved by requiring that the two sectors interact only with gravitational strength, i.e. coupling the sectors as

$$K = K^{(moduli)} + K^{(uplift)} \quad W = W^{(moduli)} + W^{(uplift)} , \quad (3.1)$$

and requesting all dimensionful couplings in the uplifting sector to be small compared to the Planck mass. However, this ansatz does not satisfy in general the necessary conditions for consistent supersymmetric decoupling of the heavy moduli (chapter 2 and de Alwis, 2005b,a, Ben-Dayan et al., 2008). For later developments on  $F$ -term uplifting mechanism see Saltman and Silverstein (2004), Lebedev et al. (2006), Lebedev et al. (2007), Dine et al. (2006), Kitano (2006), Dudas et al. (2007), Kallosh and Linde (2007a), Abe et al. (2007b), Abe et al. (2007a), Lalak et al. (2007a), Brax et al. (2007), Abe et al. (2008), Papineau (2008), Everett et al. (2008a), Covi et al. (2008b), Aparicio et al. (2008), Everett et al. (2008b), Blumenhagen et al. (2008), Abe (2009), de Alwis (2009), Jeong and Shin (2009), de Alwis (2010), Blumenhagen et al. (2009), Correia and Schmidt (2010), Badziak (2010) and Maru (2010).

In these types of models it is tempting to argue that the effects of truncating the heavy fields inconsistently would be too small to affect seriously the physics of the effective theory. Although this might be correct if we are only interested in low energy phenomenology, when the effective theory is used to describe inflation the situation is

much more subtle. In this scenario the inflationary sector is what plays the role of the supersymmetry breaking sector and, as in the case of uplifting, its interactions with the moduli fields have to be consistent with the supersymmetric integration of the heavy moduli. Recently Davis and Postma (2008) discussed an enlightening example that illustrates the problems of an inconsistent truncation in an inflationary model. They studied the  $F$ -term hybrid inflationary model proposed in Kallosh and Linde (2004) which includes a moduli sector of the KKLT or racetrack (Blanco-Pillado et al., 2004) form. This model gives viable inflation as long as the volume modulus is assumed to be fixed during inflation and some of the parameters are fine-tuned. However, this truncation of the modulus field is not consistent. When the dynamics of this field is taken into account it can be seen that the field does not remain constant during the inflationary period, it shifts from its stable value at the end of inflation. The shift results in corrections to the inflationary potential that spoil its flatness and therefore ruin inflation (see e.g. Ben-Dayan et al., 2008 for a recent discussion and references).

In the previous chapter we revisited the conditions for consistent supersymmetric decoupling of the heavy moduli. We found that these conditions can, when  $W \neq 0$ , be translated into a particular form of the Kähler invariant function  $G = K + \log |W|^2$ . For example, if the Kähler potential is separable,  $K = K^{(h)}(\text{heavy}) + K^{(l)}(\text{light})$ , it is sufficient to require that the full Kähler invariant function is also separable:

$$G = G^{(h)}(\text{heavy}) + G^{(l)}(\text{light}) , \quad (3.2)$$

or, equivalently, that the superpotential factorises in the two sectors:

$$W = W^{(h)}(\text{heavy})W^{(l)}(\text{light}) . \quad (3.3)$$

Note that consistent decoupling does not require the scalar manifold to be a product, and thus this is just a special case of the class of interactions consistent with the supersymmetric integration of the heavy fields.

The ansatz (eq. 3.2) has a long history. In the early 80's it was studied as mechanism to couple the visible matter fields to a supersymmetry breaking sector or the inflationary sector (Cremmer et al., 1983a, Binetruy and Gaillard, 1985), and more recently has been discussed as a sufficient condition to consistently truncate supersymmetric heavy chiral multiplets (Binetruy et al., 2004). It has also appeared in connection with brane inflationary models and moduli stabilisation, in particular in the  $D3/D7$  model (Hsu et al., 2003), where it was shown that the ansatz preserves the AdS flat direction (from shift symmetry) due to the BPS character of the configuration.

Achúcarro and Sousa (2008) studied the possibility of using a separable Kähler function (eq. 3.2) as an alternative way to couple the heavy moduli to the supersymmetry breaking sector in  $F$ -term uplifting mechanisms. This type of coupling ensures that if the heavy fields are truncated at a supersymmetric critical point they remain at a critical point of the potential after adding the supersymmetry breaking sector. Moreover, the  $F$ -terms of the heavy moduli remain zero after the uplifting, and thus supersymmetry breaking receives no contribution from the heavy fields.

It is remarkable that, in spite of the direct couplings present in  $W$ , the light and heavy sectors almost do not interact (Achúcarro and Sousa, 2008 and chapter 2) even when supersymmetry is broken by the light sector. Actually using the ansatz (eq. 3.2) the perturbative stability analysis of the uplifted vacuum decouples in the two sectors. In particular the stability condition along the heavy field directions has a simple form, it has no dependence on the details of the uplifting sector other than through a single parameter that measures the amount of uplifting,  $H/m_{3/2}$ , the ratio of the Hubble expansion rate to the gravitino mass of the uplifted vacuum. Achúcarro and Sousa (2008) analysed a toy model with a single heavy field and found a region in parameter space where the critical points of the heavy sector remain stable<sup>1</sup> for arbitrary values of this uplifting parameter  $H/m_{3/2}$ . Interestingly, these critical points are *stable* AdS maxima before the uplifting, and in our model correspond to minima of the Kähler function  $G^{(h)}(\text{heavy})$ . In this chapter we will prove that this result can be extended to an arbitrary number of chiral fields in the heavy supersymmetric sector, provided they satisfy (eq. 3.2), and that it survives the inclusion of gauge interactions. Also, in more general scenarios where the Kähler function is not required to be separable, we will prove using mild assumptions that the supersymmetric AdS maxima of the potential always become stable for large enough values of the uplifting parameter. The remarkable stability of dS vacua resulting from highly uplifted AdS local maxima had not been noticed before and constitutes one of the main results in this chapter.

Our work complements those Gomez-Reino and Scrucca (2006b,a, 2007) and, more recently, Covi et al. (2008b), who give necessary conditions for the stability of uplifted vacua along the supersymmetry breaking directions, which we include in the light sector. Here instead we obtain necessary and sufficient conditions for the stability of the supersymmetrically truncated moduli, about which we have little information or observational input. We take for granted the existence of stable dS vacua in the effective theory for the light sector, which therefore has to satisfy the conditions from Covi et al. (2008b). We will return to this point at the end of this

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<sup>1</sup>In this chapter, as in Achúcarro and Sousa (2008), we only study the perturbative stability of the uplifted vacua and therefore, after the uplifting, by stable vacua we mean *local* minima of the scalar potential.

chapter.

In this work we will not make any specific assumptions about the origin of the set of fields that are truncated supersymmetrically. For example in the KKLT framework they could be identified with both Kähler and complex structure moduli, or with the complex structure moduli alone depending on the masses of each set of fields. However, it should be clear that our results are not restricted to type IIB compactifications. Regarding the light sector we expect it to include both the visible sector and the hidden sector where supersymmetry is broken spontaneously.

This chapter is organised as follows. We begin in section 3.2 with a quick review of the basic features of the  $F$ -term uplifting mechanism characterised by the ansatz (eq. 3.2). In section 3.3 we study the relation between the critical points of the Kähler invariant function  $G$  and the scalar potential and prove a one-to-one correspondence between the minima of  $G$  and the supersymmetric (AdS) local maxima of the potential. The results in section 3.3 are completely general – we make no assumptions about the form of  $G$ , only that supersymmetry is unbroken. At the same time we introduce the technique used in later sections for the stability analysis of the uplifted vacua. In section 3.4 we analyse the stability of the heavy fields in  $F$ -term uplifted vacua where the coupling to the light (supersymmetry breaking) sector is given by the ansatz (eq. 3.2). We extend the results of Achúcarro and Sousa (2008) to an arbitrary (supersymmetric) heavy sector; we also consider the effect of gauge couplings and  $D$ -terms and show that the results are unchanged for consistently decoupled charged fields. Then, in section 3.5, we consider more general uplifting scenarios where the coupling between the light and heavy fields no longer satisfies (eq. 3.2) but only the milder condition  $K = K^{(h)}(\text{heavy}) + K^{(l)}(\text{light})$ . We finish with a summary of our results and a discussion in section 3.6.

## 3.2 $F$ -term uplifting and integrating out heavy scalars

Let us now review the basic features of the  $F$ -term uplifting mechanism proposed in Achúcarro and Sousa (2008). In this new class of  $F$ -term uplifting the couplings between the heavy moduli and the uplifting sector are consistent with the requirements found in the previous chapter for the supersymmetric integration of heavy moduli. The coupling between the heavy moduli,  $H^\alpha$ , and the uplifting sector, which belongs to the set of light fields  $L^i$ , is defined in terms of an ansatz for the Kähler function of the form (eq. 3.2)

$$G(H, \bar{H}, L, \bar{L}) = A(H, \bar{H}) + B(L, \bar{L}) . \quad (3.4)$$

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In the absence of gauge interactions the scalar potential derived from this ansatz can be seen to be, using (eq. 1.43)

$$V = e^{A+B} \left( A^{\alpha\bar{\beta}} A_{\alpha} A_{\bar{\beta}} + B^{i\bar{j}} B_i B_{\bar{j}} - 3 \right). \quad (3.5)$$

Fixing the heavy fields at the supersymmetric critical point  $H_0^{\alpha}$  we obtain the effective potential of the low energy theory, which reduces to the simple expression:

$$V(H_0, L) = e^{A(H_0)} V_{light}(L), \quad (3.6)$$

where  $V_{light} = e^B (B^{i\bar{j}} B_i B_{\bar{j}} - 3)$  is the scalar potential of the uplifting sector when considered alone. The uplifting properties of this ansatz can be summarised as follows:

- Suppose that  $H_0^{\alpha}$  is a supersymmetric critical point of the heavy sector, and  $L_0^i$  is a critical point of  $V_{light}$ , then the field configuration  $(H_0^{\alpha}, L_0^i)$  is a critical point of the full potential.
- The value of the potential of the light sector  $V_{light}(L)$  at the critical point  $L_0^i$  determines whether the supersymmetric vacuum is lifted to dS, Minkowski or remains AdS:

$$\begin{aligned} V_{light}(L) > 0 &\implies (H_0^{\alpha}, L_0^i) \text{ is a dS vacuum} \\ V_{light}(L) = 0 &\implies (H_0^{\alpha}, L_0^i) \text{ is a Minkowski vacuum} \\ V_{light}(L) < 0 &\implies (H_0^{\alpha}, L_0^i) \text{ is an AdS vacuum.} \end{aligned}$$

- If there is more than one supersymmetric configuration of the heavy sector, all of them become degenerate when uplifted to Minkowski (note that this makes the possibility of topological inflation quite natural).

In view of the direct couplings in (eq. 3.5) one might think that the two sectors strongly influence each other and therefore the uplifting would easily destabilise the heavy moduli, however in Achúcarro and Sousa (2008) it was found that the two sectors almost do not interact. In this paper, Achúcarro and Sousa studied the perturbative stability of vacua of the form  $(H_0^{\alpha}, L_0^i)$  where  $H_0^{\alpha}$  is a supersymmetric configuration of the heavy sector and  $L_0^i$  a critical point of  $V_{light}$ . They found that the mass matrix around this vacuum is block diagonal in the two sectors, meaning that there are no quadratic interactions between the fluctuations of the heavy and light fields,

$$\partial_{i\alpha}^2 V(H_0, L_0) = 0, \quad \partial_{i\bar{\alpha}}^2 V(H_0, L_0) = 0. \quad (3.7)$$

Actually it turns out that this is a prerequisite for freezing the heavy fields consistently. To consistently truncate the fluctuations with large masses around a given vacuum first we have to find their mass spectrum, which requires diagonalising the mass matrix, and only after having identified the heavy modes can we set them consistently to zero. Proceeding in this way, by construction, the mass matrix at the vacuum is always block diagonal in the massive and light modes.

This result allows us to study the stability of the two sectors separately. In the case of the supersymmetry breaking sector it is possible to prove a remarkably simple result, namely that the vacuum  $(H_0^\alpha, L_0^i)$  is perturbatively stable with respect to fluctuations of the light fields as long as  $L_0^i$  is a minimum of the potential of the light sector  $V_{light}(L)$ . For the heavy sector it is more difficult to obtain model independent statements concerning the perturbative stability. Nevertheless, Achúcarro and Sousa (2008) were able to give a few general results:

- the stability analysis for fluctuations of the heavy fields depends on the light sector only through a single parameter  $b = B^{i\bar{j}}B_iB_{\bar{j}}|_{L_0}$  that controls the amount of uplifting,

$$b - 3 = e^{-G} V = \left( \frac{3H}{m_{3/2}} \right)^2, \quad (3.8)$$

- any vacuum becomes stable or neutrally stable with respect to fluctuations of the heavy fields after being uplifted to Minkowski. A similar result was found in Blanco-Pillado et al. (2006a)), where it was argued that SUSY vacua with vanishing cosmological constant are automatically stable, up to flat directions,
- for large amounts of uplifting the full potential becomes approximately:

$$V(H, L) \approx b e^{A+B}, \quad (3.9)$$

and therefore the stable configurations of the heavy sector are those minimising the Kähler function  $A(H, \bar{H})$ .

In order to understand better the details of this new uplifting mechanism we analysed the perturbative stability of the supersymmetric sector in a toy model with only one heavy field. The result of this study was quite surprising, we found that the supersymmetric AdS maxima of the potential at zero uplifting ( $b = 0$ ), which are stable since they satisfy the Breitenlohner-Freedman bound, remain stable configurations of the heavy sector for any uplifting. Interestingly, we also found that these AdS maxima coincide with the minima of the Kähler function. When we studied the uplifting of AdS minima of the scalar potential we recovered the standard result, for sufficiently

large amount of uplifting these configurations always become unstable. This result opens a new door for the construction of stable dS vacua, instead of constraining ourselves to the uplifting of AdS minima we can also use the AdS maxima of the potential which seem to have better stability properties, at least for a certain class of interactions between the moduli and the uplifting sectors.

In the previous chapter we also considered the case where the gauge interactions of the light sector were turned on. We found that, as long as the heavy fields are consistently decoupled, the gauge interactions in the light sector do not change the results listed above. The consistent decoupling of the heavy sector imposes certain restrictions on the type of allowed gauge interactions. In particular any gauge field that survives at low energies should not interact simultaneously with both heavy and light fields, otherwise the heavy fields could be sourced in the low energy theory due to the gauge interactions. In practise, this requirement means that the Killing vectors of the light sector can only have components along the light directions,  $k_a^i$ . Moreover, a consistent decoupling also demands that the critical points of the heavy sector do not shift due to the presence of the light sector, implying that the Killing vectors and the gauge kinetic functions of the light sector cannot depend on the heavy fields,  $k = k_a^i(L)$ ,  $f_{ab}(L)$ . In this situation the contribution to the scalar potential generated by gauge interactions, the  $D$ -term potential (eq. 1.44), is independent of the heavy fields, and thus the stability analysis along the heavy directions is unaffected.

### **3.3 Stability of supersymmetric critical points**

In this section we study the stability properties of a supersymmetric critical point in a completely general setup. We take the action to be characterised by a Kähler potential  $G(\xi, \bar{\xi})$ , and we allow for an arbitrary gauge coupling defined by the gauge kinetic functions  $f_{ab}(\xi)$  and Killing vectors  $k(\xi)_a^i$ . We will relate the stability of the supersymmetric vacua to the curvature of the Kähler function, and in particular we will show that maxima of the scalar potential always correspond to minima of the Kähler function.

#### **3.3.1 Analysis of the Kähler function**

We begin by studying the character of the critical points of the Kähler function, which is a simple calculation and will serve us to introduce the technique we will use later in the analysis of the scalar potential. The Taylor expansion of the Kähler potential

$G(\xi^I, \xi^{\bar{I}})$  around a supersymmetric critical point  $\xi_0^I$ , reads

$$G = G(\xi_0) + G_I(\xi_0)\hat{\xi}^I + G_{\bar{I}}(\xi_0)\hat{\xi}^{\bar{I}} + G_{I\bar{J}}(\xi_0)\hat{\xi}^I\hat{\xi}^{\bar{J}} + \frac{1}{2}G_{IJ}(\xi_0)\hat{\xi}^I\hat{\xi}^J + \frac{1}{2}G_{\bar{I}\bar{J}}(\xi_0)\hat{\xi}^{\bar{I}}\hat{\xi}^{\bar{J}} + \dots, \quad (3.10)$$

where we define  $\hat{\xi}^I = \xi^I - \xi_0^I$ . Note that the first order terms vanish since  $G_I(\xi_0) = 0$ . In order to know if  $\xi_0^I$  corresponds to a minimum, a maximum or a saddle point the Kähler function we need to find the eigenvalues of its Hessian evaluated at the critical point

$$\begin{pmatrix} G_{I\bar{J}} & G_{IJ} \\ G_{\bar{I}\bar{J}} & G_{\bar{I}J} \end{pmatrix}_{\xi_0}. \quad (3.11)$$

This expression simplifies considerably by redefining the  $\xi^I$  fields so that they have canonical kinetic terms at the critical point,  $G_{IJ}(\xi_0) = \delta_{IJ}$ . With this choice of coordinates the equation for the eigenvalues,  $g$ , takes the form

$$\det \begin{pmatrix} (1-g)\mathbb{1} & X \\ X^\dagger & (1-g)\mathbb{1} \end{pmatrix} = 0, \quad (3.12)$$

where we have used the matrix notation  $X = X^T \simeq G_{IJ}(\xi_0)$  and  $\mathbb{1} \simeq \delta_{I\bar{J}}$ . Using a known property of determinants,

$$\det \begin{pmatrix} M & P \\ Q & N \end{pmatrix} = \det(MN - QP), \quad \text{provided that } QN = NQ \text{ and } \det(M) \neq 0, \quad (3.13)$$

for square submatrices  $M, N, P, Q$ , we can see that  $g$  is a solution of (eq. 3.12) if and only if it also solves

$$\det((1-g)^2 - X^\dagger X) = 0. \quad (3.14)$$

Strictly speaking this equation was derived for  $g \neq 1$ , but it is not difficult to check that it also gives the correct solution for  $g = 1$ . In order to solve this equation we use the freedom of field redefinition once more. Requiring that the fields have canonical kinetic terms is not enough to fix the choice of fields completely, we can still redefine the fields by a constant unitary transformation of the form  $\tilde{\xi}^I = U^I_J \xi^J$ . Under this field redefinition the matrix  $X$  and the combination  $X^\dagger X$  transform as

$$X = U^T \tilde{X} U, \quad X^\dagger X = U^\dagger (\tilde{X}^\dagger \tilde{X}) U, \quad \text{where } U = U^I_J, \quad (3.15)$$

and therefore we can use this freedom to transform the Hermitian combination  $X^\dagger X$  into a real diagonal matrix. The eigenvalues of  $X^\dagger X$  are necessarily nonnegative, and we will denote them by  $|x_\lambda|^2$ , with  $\lambda$  labelling the  $p$  different eigenspaces. Moreover,

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the symmetry of  $X$  implies that  $X^\dagger X = (XX^\dagger)^*$ , thus in the basis that makes  $X^\dagger X$  diagonal we have

$$X^\dagger X = XX^\dagger = \text{Diag}(|x_1|^2 \mathbb{1}_{n_1}, \dots, |x_p|^2 \mathbb{1}_{n_p}), \quad |x_\lambda|^2 \geq 0, \quad (3.16)$$

where  $n_\lambda$  is the dimension of the eigenspace of eigenvalue  $|x_\lambda|^2$ . Note also that in this particular basis the matrices  $X$  and  $X^\dagger X$  commute, which implies that  $X$  should be block diagonal in each of the eigenspaces of  $X^\dagger X$ :

$$X = \text{Diag}(X_1, \dots, X_p) \quad \text{with} \quad X_\lambda^\dagger X_\lambda = |x_\lambda|^2 \mathbb{1}_{n_\lambda}. \quad (3.17)$$

This means that the eigenvalue problem decouples for the  $m$  different eigenspaces of  $X^\dagger X$ , and therefore the equation (eq. 3.14) takes a very simple form

$$\prod_{\lambda=1}^p [(1 - g_\lambda)^2 - |x_\lambda|^2]^{n_\lambda} = 0, \quad (3.18)$$

which we can solve easily giving the eigenvalues

$$g_{\pm\lambda} = 1 \pm |x_\lambda|. \quad (3.19)$$

which have multiplicity  $n_\lambda$ . The different possibilities for the character of the critical point  $H_0^\alpha$  are summarised in the following table:

$$\begin{aligned} \text{Local minimum} & |x_\lambda| < 1 \quad \text{for all } \lambda = 1, \dots, p, \\ \text{Saddle point} & |x_\lambda| > 1 \quad \text{for some or all } \lambda. \end{aligned} \quad (3.20)$$

The result (eq. 3.19) also indicates that, for each eigenvalue of  $X^\dagger X$  that satisfies  $|x_\lambda|^2 = 1$ , the Kähler function has a flat direction and a local minimum (a trough) along one of the complex directions  $\xi^l$ .

### 3.3.2 Analysis of the scalar potential with vanishing $D$ -terms

We will now analyse how the maxima and saddle points of the Kähler function relate to the different types of supersymmetric critical points of the scalar potential. This is especially interesting because the Kähler function is much easier to study. We will demonstrate a remarkable result: the minima of the Kähler function are in one to one correspondence with the supersymmetric AdS maxima of the scalar potential. We start assuming that there are no gauge interactions, and in the next section we will prove this result in full generality.

The stability analysis of a supersymmetric critical point of the scalar potential can be done using the same techniques of the previous subsection. Consider first its Taylor expansion around a supersymmetric critical point  $\xi_0^I$ ,

$$V = V(\xi_0) + \frac{1}{2} V_{IJ}(\xi_0) \hat{\xi}^I \hat{\xi}^J + \frac{1}{2} V_{\bar{I}\bar{J}}(\xi_0) \hat{\xi}^{\bar{I}} \hat{\xi}^{\bar{J}} + V_{IJ}(\xi_0) \hat{\xi}^I \hat{\xi}^{\bar{J}} + \dots, \quad (3.21)$$

where the second derivatives of the potential evaluated at the point  $\xi^I = \xi_0^I$  can be calculated from (eq. 1.43) using that  $G_I(\xi_0) = 0$ ,

$$V_{IJ}(\xi_0) = -G_{IJ}(\xi_0) e^{G(\xi_0)}, \quad (3.22)$$

$$V_{\bar{I}\bar{J}}(\xi_0) = e^{G(\xi_0)} \left[ G^{R\bar{S}} G_{RI} G_{S\bar{J}} - 2G_{I\bar{J}} \right]_{\xi_0}. \quad (3.23)$$

In order to determine the character of the critical point  $\xi_0^I$  we need to find the eigenvalues of the Hessian of the potential, which gives the mass spectrum of the fluctuations around  $\xi_0^I$ . As in the previous subsection we define the fields  $\xi^I$  so that they have canonical kinetic terms,  $G_{I\bar{J}}(\xi_0) = \delta_{I\bar{J}}$ , and the Hermitian matrix  $X^\dagger X$  becomes diagonal. With this choice the Hessian has the simple form

$$\begin{pmatrix} V_{IJ} & V_{I\bar{J}} \\ V_{\bar{I}\bar{J}} & V_{\bar{I}J} \end{pmatrix}_{\xi_0} = e^{G(\xi_0)} \begin{pmatrix} XX^\dagger - 2\mathbb{1} & -X \\ -X^\dagger & X^\dagger X - 2\mathbb{1} \end{pmatrix}. \quad (3.24)$$

Since in the basis we have chosen  $X^\dagger X = XX^\dagger$  it is easy to check that this matrix also satisfies the first of the conditions necessary to apply (eq. 3.13), and we find that the equation for the spectrum of masses  $m^2$  reads

$$\det \left( (X^\dagger X - (2 + e^{-G(\xi_0)} m^2) \mathbb{1})^2 - X^\dagger X \right) = 0. \quad (3.25)$$

In order to use (eq. 3.13) we also need to assume that the following matrix is non-singular,

$$\det \left( X^\dagger X - (2 + e^{-G(\xi_0)} m^2) \mathbb{1} \right) \neq 0, \quad (3.26)$$

but after some algebra it is possible to prove that (eq. 3.25) also gives the right result in the singular case. As in the previous section we can use that  $X$  has the block diagonal form (eq. 3.17) to show that the eigenvalue problem can be decomposed in each of the eigenspaces of  $X^\dagger X$ . Using this fact the eigenvalue equation (eq. 3.25) can be written as

$$\prod_{\lambda=1}^p \left[ (|x_\lambda|^2 - 2 - e^{-G(\xi_0)} m^2)^2 - |x_\lambda|^2 \right]^{n_\lambda} = 0. \quad (3.27)$$

Therefore the spectrum of masses at the supersymmetric critical point is given by

$$m_{\pm\lambda}^2 = e^{G(\xi_0)}(|x_\lambda|^2 - 2 \pm |x_\lambda|) = e^{G(\xi_0)} \left( \left( |x_\lambda| \pm \frac{1}{2} \right)^2 - \frac{9}{4} \right). \quad (3.28)$$

Each of these energy levels contains  $n_\lambda$  different excitations with the same mass. The set of quantities  $|x_\lambda|$  determine which type of extremum the supersymmetric critical point  $\xi_0^I$  is

$$\begin{aligned} |x_\lambda| > 2 & \quad \text{for all } \lambda \Rightarrow \text{local AdS minimum,} \\ |x_\lambda| < 1 & \quad \text{for all } \lambda \Rightarrow \text{local AdS maximum,} \end{aligned} \quad (3.29)$$

and any other combination corresponds to AdS saddle points ( $|x_\lambda| = 1, 2$  give flat directions). The result (eq. 3.28) also provides a proof of the stability of all supersymmetric critical points, regardless of the possible negative curvature of the potential. Since all these critical points are AdS, the perturbative stability is determined by the Breitenlohner-Freedman bound (eq. 1.53), which is always satisfied as is clear from (eq. 3.28),

$$m^2 \geq -\frac{9}{4} e^{G(\xi_0)} = \frac{3}{4} V(\xi_0). \quad (3.30)$$

Now we already have at hand all the results we need to check the claim we made at the beginning of this subsection. Comparing equations (eq. 3.20) and (eq. 3.29) we see immediately that the supersymmetric AdS maxima of the potential always coincide with the minima of the Kähler function.

### 3.3.3 Analysis of the scalar potential with non-vanishing $D$ -terms

Let us now generalise the result of the previous subsection to the case where the gauge couplings are turned on. In this case we have to add to the scalar potential the contribution from  $D$ -terms (eq. 1.46). In order to calculate the new contributions to the Hessian of the scalar potential around the critical point  $\xi_0^I$  we must find the derivatives of the  $D$ -term potential at this point. Using that  $G_I(\xi_0) = 0$  we find

$$\begin{aligned} V_{D|IJ}(\xi_0) &= \frac{1}{2} (\text{Re}f(\xi_0))^{-1ab} k_a^R(\xi_0) k_b^{\bar{S}}(\bar{\xi}_0) [G_{IR}G_{J\bar{S}} + G_{JR}G_{I\bar{S}}]_{\xi_0}, \\ V_{D|I\bar{J}}(\xi_0) &= \frac{1}{2} (\text{Re}f(\xi_0))^{-1ab} k_a^R(\xi_0) k_b^{\bar{S}}(\bar{\xi}_0) [G_{IR}G_{\bar{J}\bar{S}} + G_{\bar{J}R}G_{I\bar{S}}]_{\xi_0}. \end{aligned} \quad (3.31)$$

As we have done previously we will define the scalar fields  $\xi^I$  so that they have trivial kinetic terms  $G_{IJ} = \delta_{IJ}$  and the Hermitian matrix  $X^\dagger X$  is diagonal. In the case of the  $D$ -term potential we can simplify the calculations even further making use of the

freedom we have to define the gauge fields  $A_\mu^a$ . Actually, the action is invariant under constant linear transformations of the gauge fields  $A_\mu^b = O^b_a \tilde{A}_\mu^a$ , with  $O^b_a$  any non-singular real matrix, provided that the gauge kinetic functions  $f_{ab}$  and the Killing vectors  $k^I_a$  transform as

$$\tilde{f}_{cd} = O_c^a O_d^b f_{ab}, \quad \tilde{k}^I_b = O_b^a k^I_a. \quad (3.32)$$

In particular, note that the gauge covariant derivatives and the Yang-Mills terms do not transform under these redefinitions, since

$$A^a_\mu k^I_a = \tilde{A}^b_\mu \tilde{k}^I_b, \quad (\text{Re} f_{ab}) F^a_{\mu\nu} F^{a\mu\nu} = (\text{Re} \tilde{f}_{cd}) \tilde{F}^c_{\mu\nu} \tilde{F}^{d\mu\nu}. \quad (3.33)$$

We can use this freedom to turn  $\text{Re}(f_{ab})$  into a matrix proportional to the identity

$$\text{Re}(f_{ab}) = e^{G(\xi_0)} \delta_{ab}, \quad (3.34)$$

where the overall factor has been chosen for convenience. Using these conventions, and defining the matrix  $k = k^I_a(\xi_0)$ , we can write the Hessian of the  $D$ -term potential as

$$\begin{pmatrix} V_{D|IJ} & V_{D|I\bar{J}} \\ V_{D|\bar{I}\bar{J}} & V_{D|\bar{I}J} \end{pmatrix}_{\xi_0} = \frac{1}{2} e^{G(\xi_0)} \begin{pmatrix} kk^\dagger + Xkk^\dagger X^\dagger & Xkk^\dagger + k^* k^T X \\ X^\dagger k^* k^T + kk^\dagger X^\dagger & k^* k^T + X^\dagger k^* k^T X \end{pmatrix}, \quad (3.35)$$

which has to be added to (eq. 3.24) in order to get the Hessian of the full scalar potential.

Before we continue the calculation, let us derive a useful property of the Killing vectors  $k$ . We mentioned in section 1.3.2 that the Kähler function  $G(\xi^I)$  has to be invariant under gauge transformations. In particular in a Taylor expansion of the Kähler function around  $\xi_0$  (eq. 3.10) all the terms have to be invariant under gauge transformations order by order in  $\hat{\xi} = \xi - \xi_0$ . From the gauge transformation of the order one terms in the expansion we find the condition

$$\left( G_{IJ}(\xi_0) k^J_a(\xi_0) \hat{\xi}^I + G_{I\bar{J}}(\xi_0) k^{\bar{J}}_a(\xi_0) \hat{\xi}^{\bar{I}} + G_I(\xi_0) \partial_J k^I_a \hat{\xi}^J \right) \alpha^a = 0, \quad (3.36)$$

which has to be satisfied for all values of the gauge parameters  $\alpha^a$ , and the fluctuations  $\hat{\xi}^I$ . Since  $G_I(\xi_0) = 0$ , then with our field definitions and in matrix notation this condition simply reads

$$k^* = -Xk. \quad (3.37)$$

An immediate consequence of this requirement is that the Killing vectors are eigenvectors of the matrix  $X^\dagger X$  with eigenvalue  $g_\lambda = 1$ ,

$$X^\dagger X k = -X^\dagger k^* = k, \quad (3.38)$$

since  $X^T = X$ . This means that the matrices  $kk^\dagger$  and  $kk^T$  have all the entries zero except in the block that corresponds to the eigenspace of eigenvalue  $|x_\lambda|^2 = 1$  of  $X^\dagger X$ . As we saw in the previous section the eigenvalue problem of the Hessian decouples in the different eigenspaces of the matrix  $X^\dagger X$ . We have just proven that the corrections introduced by the  $D$ -term potential respect this decoupling and moreover, that the corrections only affect the eigenspace with eigenvalue  $|x_\lambda|^2 = 1$ . Therefore we can use the results of the previous section for all the eigenspaces with  $|x_\lambda| \neq 1$  to find the corresponding eigenvalues. In the remainder of this section we will just focus on solving the eigenvalue problem in the eigenspace where  $|x_\lambda|^2 = 1$ , which we label by  $\lambda = 1$ . In order to keep notation simple, we will use the matrices  $X_1$  and  $k_1$  to represent the submatrices corresponding to the eigenspace  $\lambda = 1$ , thus

$$X_1^\dagger X_1 = X_1 X_1^\dagger = \mathbb{1}_{n_1} . \quad (3.39)$$

Notice that, since the Hermitian matrix  $k_1 k_1^\dagger$  transforms under scalar field redefinitions in the same way as  $X_1 X_1^\dagger$ , we can use the residual freedom to choose the eigenvectors in the eigenspace with  $|x_\lambda|^2 = 1$  to turn  $k_1 k_1^\dagger$  into a real diagonal matrix

$$k_1 k_1^\dagger = \text{Diag}(|k_1|^2, \dots, |k_{n_1}|^2) , \quad (3.40)$$

where  $n_1$  is the dimension of the eigenspace with  $|x_\lambda|^2 = 1$ . The final expression for the Hessian of the full potential restricted to this eigenspace can be obtained from (eq. 3.35) and (eq. 3.24), and is

$$\left( \begin{array}{cc} V_{I\bar{J}} & V_{IJ} \\ V_{I\bar{J}} & V_{IJ} \end{array} \right)_{\lambda=1} = e^{G(\xi_0)} \left( \begin{array}{cc} -\mathbb{1} + k_1 k_1^\dagger & (-\mathbb{1} + k_1 k_1^\dagger) X_1 \\ (-\mathbb{1} + k_1 k_1^\dagger) X_1^\dagger & -\mathbb{1} + k_1 k_1^\dagger \end{array} \right) , \quad (3.41)$$

where we have used the properties (eq. 3.37), (eq. 3.39) and (eq. 3.40) in order to simplify (eq. 3.35). It is easy to check that the matrices  $X_1$  and  $k_1 k_1^\dagger$  commute, therefore we can use (eq. 3.13) in order to find the equation for the mass spectrum  $m^2$ , which reads

$$\prod_{i=1}^{n_1} \left( (|k_i|^2 - 1 - e^{-G(\xi_0)} m^2)^2 - (|k_i|^2 - 1)^2 \right) = 0 , \quad (3.42)$$

after having substituting the diagonal form of  $k_1 k_1^\dagger$  (eq. 3.40). The solution to this equation, together with the results we found in the previous section, which apply for

$|x_\lambda| \neq 1$  are summarised below

$$\begin{aligned}
 m_{\pm\lambda}^2 &= e^{G(\xi_0)} \left( (|x_\lambda \pm \frac{1}{2}|)^2 - \frac{9}{4} \right) && \text{if } |x_\lambda|^2 \neq 1, \\
 m_{+1i}^2 &= 2e^{G(\xi_0)} (|k_i|^2 - 1) && \text{if } |x_\lambda|^2 = 1, \\
 m_{-1i}^2 &= 0 && \text{if } |x_\lambda|^2 = 1.
 \end{aligned} \tag{3.43}$$

The quantities  $|k_i|^2$  determine the mass of the gauge bosons at the supersymmetric critical point  $\xi_0$ , therefore we can see that the breaking of gauge symmetries can only improve the stability of vacuum. This result agrees with the analysis in Gomez-Reino and Scrucca (2007) of the stability of uplifted vacua.

The fact that the Killing vectors are associated to the eigenvalues  $|x_\lambda|^2 = 1$  should not be surprising. On the one hand the eigenvalues  $|x_\lambda|^2 = 1$  are always related to marginally stable directions  $m^2 = 0$ . On the other hand we know that the potential has to be invariant under gauge transformations, thus each Killing vector has to be naturally associated with a flat direction of the potential, which appear in the spectrum as massless fluctuations (the would-be Goldstone bosons that disappear due to the Higgs mechanism).

In view of the result (eq. 3.43) we can argue that the presence of non-vanishing gauge couplings does not modify the conclusion of the previous section, namely that the minima of the Kähler function  $G(\xi^I, \xi^{\bar{I}})$  are always in one to one correspondence with the supersymmetric AdS maxima of the scalar potential.

## 3.4 Stability of uplifted vacua

We now return to the main question in this chapter, the perturbative stability of the heavy sector when these AdS supersymmetric vacua are uplifted to dS by supersymmetry breaking effects in the light sector. In this section we again assume that the Kähler invariant function is separable in the heavy and light sectors, (eq. 3.4).

### 3.4.1 Stability of uplifted vacua with zero $D$ -term potential

We start by generalising the results of Achúcarro and Sousa (2008) to an arbitrary number of heavy fields in the supersymmetric sector. We assume all fields are uncharged.

Suppose that the set of chiral fields  $\xi^I$  can be split in two sectors,  $n_h$  heavy fields  $H^\alpha$  and  $n_l$  light fields  $L^i$ , which are coupled as in (eq. 3.4). We will assume that the

system is stabilised at a critical point of the potential  $(H_0^\alpha, L_0^i)$ , which is also a supersymmetric configuration of the heavy sector,  $G_\alpha(H_0, L_0) = 0$ , but supersymmetry is broken in the light sector,  $G_i(H_0, L_0) \neq 0$ . Then, as discussed in section 3.2, the Hessian of the full potential has a block diagonal form in the two sectors (eq. 3.7) and therefore it is consistent to consider the stability of the potential only along the “heavy” and “light” directions independently. We will take the light sector fixed at a perturbatively stable configuration, and we will focus on the stability analysis along the heavy directions. For this purpose we only need to calculate  $V_{\alpha\bar{\beta}}(H_0, L_0)$  and  $V_{\alpha\beta}(H_0, L_0)$  from (eq. 3.5),

$$\begin{aligned} V_{\alpha\bar{\beta}}(H_0, L_0) &= e^{A+B}|_{H_0, L_0} \left[ A^{\gamma\bar{\delta}} A_{\alpha\gamma} A_{\beta\bar{\delta}} + (b-2) A_{\alpha\bar{\beta}} \right]_{H_0}, \\ V_{\alpha\beta}(H_0, L_0) &= e^{A+B}|_{H_0, L_0} (b-1) A_{\alpha\beta}(H_0), \end{aligned} \quad (3.44)$$

where we are using the notation  $b = B^{i\bar{j}} B_i B_{\bar{j}}|_{L_0}$ . We will use same choice of fields as in the previous section, where  $H^\alpha$  are canonically normalised at  $H_0^\alpha$  and the matrix  $X^\dagger X$  is real and diagonal, with  $X = A_{\alpha\beta}(H_0)$ . With this choice we obtain the following expression for the Hessian of the potential at  $(H_0^\alpha, L_0^i)$ :

$$\begin{pmatrix} V_{\alpha\bar{\beta}} & V_{\alpha\beta} \\ V_{\bar{\alpha}\beta} & V_{\alpha\beta} \end{pmatrix}_{H_0, L_0} = e^{A+B}|_{H_0, L_0} \begin{pmatrix} XX^\dagger + (b-2)\mathbb{1} & (b-1)X \\ (b-1)X^\dagger & X^\dagger X + (b-2)\mathbb{1} \end{pmatrix}. \quad (3.45)$$

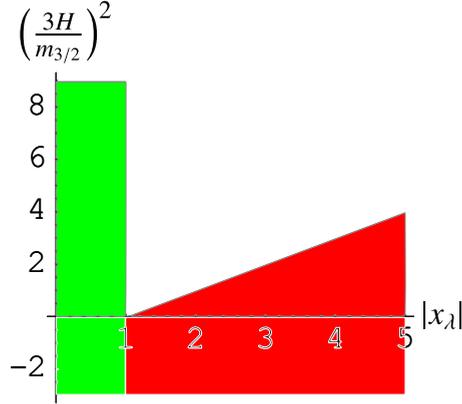
Calculating the mass spectrum as in the previous section we arrive at our final result,

$$\begin{aligned} m_{\pm\lambda}^2 &= e^{A+B}|_{H_0, L_0} \left[ |x_\lambda|^2 + (b-2) \pm |(b-1)x_\lambda| \right] \\ &= e^{A+B}|_{H_0, L_0} \left[ \left( |x_\lambda| \pm \frac{1}{2}(b-1) \right)^2 - \frac{1}{4}(b-3)^2 \right]. \end{aligned} \quad (3.46)$$

To obtain the last expression we assumed that  $b > 1$ , but in the case  $b < 1$  then  $m_{+\lambda}^2$  and  $m_{-\lambda}^2$  have to be exchanged. For each energy level characterised by  $m_{\pm\lambda}^2$  there are  $n_\lambda$  different excitations with the same mass, where, in analogy with the previous sections,  $n_\lambda$  represents the dimension of the eigenspace of  $X^\dagger X$  with eigenvalue  $|x_\lambda|^2$ . The stability condition after uplifting the minimum of the potential to Minkowski or de Sitter,  $b \geq 3$ , reduces to  $m_{\pm\lambda}^2 > 0$  for all  $\lambda = 1, \dots, p$ , but if the minimum remains AdS after the uplifting,  $b < 3$ , the masses have to satisfy the Breitenlohner-Freedman bound (eq. 1.53), which yields

$$\begin{aligned} \text{for } b < 3 &\implies \left[ \left( |x_\lambda| \pm \frac{1}{2}(b-1) \right)^2 - \frac{1}{4}(b-3)^2 \right] \geq \frac{3}{4}(b-3), \\ \text{for } b \geq 3 &\implies \left[ \left( |x_\lambda| \pm \frac{1}{2}(b-1) \right)^2 - \frac{1}{4}(b-3)^2 \right] \geq 0. \end{aligned} \quad (3.47)$$

Recalling that  $b \geq 0$ , and after a little bit of algebra, it is possible to see that the first of the two inequalities is always satisfied. The second shows that there are no instabilities when the minimum is uplifted to Minkowski  $b = 3$ , although zero modes are possible if any  $|x_\lambda| = 1$ . Thus, the instabilities can only arise for upliftings to dS. These results are summarised in figure 3.1.



**Figure 3.1:** Stability of supersymmetric critical points after the uplifting. The quantity on the vertical axis  $(3H/m_{3/2})^2$ , the Hubble parameter during inflation, is proportional to the effective cosmological constant via (eq. 3.8). The horizontal axis represents the curvature of the Kähler function, as defined in (eq. 3.16), at the critical point along one of the heavy field directions  $H^\alpha$ :  $|x_\lambda| < 1$  corresponds to local minima and  $|x_\lambda| > 1$  to saddle points. The coloured regions represent stable configurations under perturbations of the heavy fields. For  $(3H/m_{3/2})^2 < 0$  and  $(3H/m_{3/2})^2 = 0$  the uplifted vacua, which are AdS and Minkowski respectively, are always stable. Local AdS minima of the scalar potential at zero uplifting,  $|x_\lambda| > 1$ , are stable only in the red region and are always destabilised for large uplifting. Local AdS maxima,  $|x_\lambda| < 1$ , are depicted by the green region and remain stable for arbitrary large uplifting.

We can see that all the results that were obtained in the study of a single modulus toy model in Achúcarro and Sousa (2008) can be generalised to an arbitrary number of fields in the heavy sector. Local AdS minima and saddle points before the uplifting are only stable for small values of the cosmological constant, while local AdS maxima of the potential, which coincide with the local minima of the Kähler function, are always stable.

### 3.4.2 Stability of uplifted vacua with a non-zero $D$ -term potential

Now we study the stability of uplifted vacua when the gauge couplings are turned on. Including gauge interactions is specially relevant in the case of the light sector, since it includes the visible sector. In section 3.2 we mentioned that any gauge field that appears in the effective theory cannot be coupled to the fields that are consistently truncated, otherwise the gauge fields could act as sources for the truncated fields leading to an inconsistency. For instance, if the truncated fields acquire an expectation value, any gauge field coupled to it would be Higgsed and the full massive vector multiplet would have to be truncated as well. In our analysis, we will assume that Killing vectors and the gauge kinetic functions satisfy the requirements for a consistent decoupling of the heavy sector described in section 3.2. Moreover, we will ask the gauge fields in the heavy and light sectors to have decoupled kinetic terms, or in other words, that the gauge kinetic function  $f_{ab}$  is block diagonal in the two sectors. Other than that we will allow the Killing vectors and the gauge kinetic functions of the heavy sector to have an arbitrary dependence on the light and heavy fields  $k_a^\alpha(H, L)$  and  $f_{ab}^{(h)}(H, L)$ . Under these requirements the  $D$ -term potential reads

$$V_D = \frac{1}{2}(\text{Ref}(L))^{-1ab} k_a^i(L) k_b^{\bar{j}}(\bar{L}) G_i G_{\bar{j}} + \frac{1}{2}(\text{Ref}(H, L))^{-1ab} k_a^\alpha(H, L) k_b^{\bar{\beta}}(\bar{H}, \bar{L}) G_\alpha G_{\bar{\beta}} . \quad (3.48)$$

Since the gauge kinetic functions and the Killing vectors of the light sector depend only on the light fields, and  $G_i(H, L) = A_i(L)$ , all the dependence of  $V_D$  on the heavy fields comes from the second term in (eq. 3.48). Therefore, using that  $G_\alpha(H_0) = 0$ , it is easy to check that, despite the dependence of the  $D$ -term potential of the heavy sector on the light fields, the critical points of the heavy sector are preserved. Moreover, the stability analysis along the heavy field directions is also unaffected by the light sector. For example, the Hessian of the potential at the critical point remains block diagonal in the two sectors,  $V_{D|\alpha i}(H_0, L_0) = V_{D|\bar{\alpha} \bar{i}}(H_0, \bar{L}_0) = 0$ , and the second derivatives of the  $D$ -term potential along the heavy directions are given by

$$V_{D|\alpha\beta}(H_0, L_0) = \frac{1}{2}(\text{Ref}(H_0, L_0))^{-1ab} k_a^\gamma(H_0, L_0) k_b^{\bar{\delta}}(\bar{H}_0, \bar{L}_0) G_{\gamma\alpha} G_{\bar{\delta}\beta} , \\ V_{D|\alpha\bar{\beta}}(H_0, L_0) = \frac{1}{2}(\text{Ref}(H_0, L_0))^{-1ab} k_a^\gamma(H_0, L_0) k_b^{\bar{\delta}}(\bar{H}_0, \bar{L}_0) G_{\gamma\alpha} G_{\bar{\delta}\bar{\beta}} . \quad (3.49)$$

Thus, in order to find the mass matrix of the heavy fields at the critical point  $(H_0^\alpha, L_0^i)$ , we only have to add the Hessian of the  $D$ -term potential with respect to the heavy fields to the result we found for the  $F$ -term potential (eq. 3.45). We choose

our heavy scalar fields so that they have trivial kinetic terms  $G_{\alpha\bar{\beta}} \simeq \mathbb{1}^2$  and that the matrices  $X^\dagger X$  ( $X = G_{\alpha\beta}$ ) and  $k^\alpha_a k^\beta_a$  are both real and diagonal. We also define the gauge fields  $A_\mu^{(h)a}$  such that the real parts of the gauge kinetic functions of the heavy sector are proportional to the identity matrix,

$$\text{Re} f_{ab}^{(h)} = e^G|_{H_0, L_0} \delta_{ab} . \quad (3.50)$$

The calculation of the  $D$ -term contribution to the Hessian can be done along the same lines as in section (eq. 3.4.2). Is not difficult to check that the properties (eq. 3.37) and (eq. 3.38) still hold, thus here again the matrices  $kk^\dagger$  and  $kk^T$ , with  $k = k^\alpha_a$ , have non-vanishing components only in the eigenspace corresponding to the eigenvalue  $|x_\lambda|^2 = 1$ . Thus, after some simplifications, the Hessian of the total scalar potential reads

$$\begin{pmatrix} V_{\alpha\bar{\beta}} & V_{\alpha\beta} \\ V_{\bar{\alpha}\bar{\beta}} & V_{\bar{\alpha}\beta} \end{pmatrix}_{H_0, L_0} = e^{A+B}|_{H_0, L_0} \begin{pmatrix} XX^\dagger + kk^\dagger + (b-2)\mathbb{1} & (b-1 + kk^\dagger)X \\ (b-1 + kk^\dagger)X^\dagger & X^\dagger X + kk^\dagger + (b-2)\mathbb{1} \end{pmatrix} . \quad (3.51)$$

From this expression it is straightforward to find the mass spectrum of fluctuations of the heavy sector along the heavy directions:

$$\begin{aligned} m_{\pm\lambda}^2 &= e^{G(\xi_0)} \left( (|x_\lambda \pm \frac{1}{2}(b-1)|)^2 - \frac{1}{4}(b-3)^2 \right) && \text{if } |x_\lambda|^2 \neq 1, \\ m_{+1i}^2 &= 2 e^{G(\xi_0)} (|k_i|^2 + b - 1) && \text{if } |x_\lambda|^2 = 1, \\ m_{-1i}^2 &= 0 && \text{if } |x_\lambda|^2 = 1. \end{aligned} \quad (3.52)$$

We can see that figure 3.1 is still valid when we include the gauge interactions. The only difference with the result in the previous section is that if some of the gauge symmetries are spontaneously broken the mass degeneracy in the eigenspace with  $|x_\lambda| = 1$  is destroyed. From (eq. 3.52) we can see that the presence of gauge interactions only increases the stability of the critical point.

## 3.5 More general couplings

An interesting question to consider is whether it is possible apply our results to other systems where light and heavy moduli are not coupled with the ansatz (eq. 3.4). Let

<sup>2</sup>We do not consider the possibility of a nilpotent Kähler metric (Groot Nibbelink et al., 2001). We thank Jan-Willem van Holten for pointing this out.

### Chapter 3: *F*-term uplifting and the supersymmetric integration of heavy scalars

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us assume only the mild condition that the Kähler potentials are separable in the light and heavy sectors,

$$K = K^h(H, \bar{H}) + K^l(L, \bar{L}) .$$

This condition ensures that mixed derivatives of the Kähler function of the form  $G_{i\bar{\alpha}}(H_0, L_0)$ ,  $G_{i\bar{\alpha}\beta}(H_0, L_0)$  et cetera, involving both holomorphic and antiholomorphic indices from the two sectors, must vanish. We will also focus on cases where supersymmetry is unbroken at low energies, thus we take the heavy fields fixed at a supersymmetric critical point.

As we discussed in section 3.2, the condition that the potential has to be block diagonal in the light and heavy fields is necessary in order to truncate the heavy fields consistently. Therefore, in any scenario where part of the moduli are going to be truncated the stability of these fields can be studied independently considering only the “heavy” directions in field space. In order to recover a mass matrix of the form (eq. 3.44) and (eq. 3.51) we would need to satisfy the extra condition

$$G_{i\alpha}(H_0, L_0) = 0 . \quad (3.53)$$

We can prove this equation from the requirement that the low energy effective action must be invariant under supersymmetry transformations. In particular, this requirement means that the supersymmetry transformations cannot generate the fields that we have truncated. Consider the supersymmetry transformation of the fermions of the heavy sector, which in a homogeneous bosonic background are simply

$$\delta\chi^\alpha_L = -\frac{1}{2}e^{\frac{\kappa}{2}}\bar{W}G^{\alpha\bar{\beta}}G_{\bar{\beta}}\epsilon_L . \quad (3.54)$$

Expanding this expression to first order in the fluctuations of the light fields around the critical point  $L^i = L_0^i + \hat{L}^i$  gives

$$\delta\chi^\alpha_L = -\frac{1}{2}e^{\frac{\kappa}{2}}\bar{W}G^{\alpha\bar{\beta}}G_{\bar{\beta}}(H_0, L_0)\hat{L}^i\epsilon_L . \quad (3.55)$$

We can see that, unless the quantity  $G_{i\alpha}(H_0, L_0)$  vanishes, the supersymmetry transformations will generate the fermions of the heavy sector ( $W \neq 0$ ). The condition (eq. 3.53) also ensures that the point where we are fixing the heavy moduli is an extremum of the scalar potential,

$$V_\alpha(H_0, L_0) = \left[ e^G G^{i\bar{j}} G_{i\alpha} G_{\bar{j}} \right]_{H_0, L_0} = 0 , \quad (3.56)$$

where we have already used that the Kähler potential is separable and  $G_\alpha(H_0, L_0) = 0$ .

Since all the mixed derivatives of the Kähler potential  $G_{i\alpha}(H_0, L_0)$  and  $G_{i\bar{\alpha}}(H_0, L_0)$  vanish, it makes sense to study the curvature of  $G(H, L)$  at the critical point  $H_0^\alpha$  only

along the heavy directions. Thus, we can repeat the analysis of section 3.1 arriving at similar conclusions:

- the Kähler function  $G(L, H)$  has a local minimum at  $H_0^\alpha$  along the heavy directions if the eigenvalues of the matrix  $X^\dagger X$  satisfy the conditions  $|x_\lambda| < 1$  for all  $\lambda = 1, \dots, p$ , with  $X = G_{\alpha\beta}(H_0, L_0)$ ,
- if any of the eigenvalues of  $X^\dagger X$  satisfies  $|x_\lambda| > 1$  the function  $G(H, L)$  has a saddle point at  $H_0$ ,
- for each eigenvalue of  $X^\dagger X$  satisfying  $|x_\lambda| = 1$  the Kähler function has a neutrally stable direction and a minimum along some complex direction  $H^\alpha$ .

Using all these results, we can now study the stability of the scalar potential along the heavy directions as in section 3.2. The second derivatives of the scalar potential are given by

$$V_{\alpha\beta}(H_0, L_0) = e^G|_{H_0, L_0} \left[ (b-1)G_{\alpha\beta} + G^{i\bar{j}}G_{i\alpha\beta}G_{\bar{j}} \right]_{H_0, L_0}, \quad (3.57)$$

$$V_{\alpha\bar{\beta}}(H_0, L_0) = e^G|_{H_0, L_0} \left[ G^{\gamma\bar{\delta}}G_{\alpha\gamma}G_{\bar{\beta}\bar{\delta}} + (b-2)G_{\alpha\bar{\beta}} \right]_{H_0, L_0}, \quad (3.58)$$

where we have used the notation  $b = G^{i\bar{j}}G_i G_{\bar{j}}|_{H_0, L_0}$ .

Note that, apart from the second term in the equation (eq. 3.57), the result we have obtained is of the same form as (eq. 3.44). If the quantity  $G_{i\alpha\beta}$  stays of order  $\mathcal{O}(1)$ , the extra term that we have obtained is roughly of order  $\mathcal{O}(b^{1/2})$ , which means that for large values of the uplifting,  $b \gg 3$ , it will become subdominant. Therefore, in this limit, the mass matrix becomes proportional to the Hessian of the Kähler function at  $H_0^\alpha$ ,

$$\begin{pmatrix} V_{\alpha\bar{\beta}} & V_{\alpha\beta} \\ V_{\bar{\alpha}\bar{\beta}} & V_{\bar{\alpha}\beta} \end{pmatrix}_{H_0, L_0} = b \begin{pmatrix} \mathbb{1} & X \\ X^\dagger & \mathbb{1} \end{pmatrix} e^{A+B}|_{H_0, L_0} = b \begin{pmatrix} G_{\alpha\bar{\beta}} & G_{\alpha\beta} \\ G_{\bar{\alpha}\bar{\beta}} & G_{\bar{\alpha}\beta} \end{pmatrix}_{H_0, L_0} e^{A+B}|_{H_0, L_0}, \quad (3.59)$$

indicating that the minima of the Kähler function along the heavy directions will always survive uplifting to an arbitrary large value of the cosmological constant. Note also that before uplifting,  $G_i(H_0, L_0) = 0$ , the mass matrix given by (eq. 3.58) coincides with (eq. 3.23), so we can again identify the AdS maxima of the scalar potential with the local minima of the Kähler function along the heavy directions.

We would like to emphasise that in order to obtain this result we have made very mild assumptions. We have required that the Kähler potential is separable in the two sectors, we have also imposed the condition that the effective action left after truncating the heavy moduli is invariant under supersymmetry, and finally we asked

the quantity  $G_{i\alpha\beta}$  to stay of order  $\mathcal{O}(1)$  for large values of the uplifting. In this scenario we have proved that the AdS maxima of the potential along the heavy directions at zero uplifting ( $G_i(H_0, L_0) = 0$ ), which are perturbatively stable configurations, remain stable after the uplifting for arbitrary large values of the cosmological constant.

### 3.6 Summary and Conclusions

In this chapter we have studied in detail the stability properties of the  $F$ -term uplifting mechanism recently proposed in Achúcarro and Sousa (2008). This way of uplifting AdS vacua guarantees that the interactions between the uplifting sector and the moduli of the compactification are consistent with supersymmetrically truncating the heavy fields (chapter 2). The exact composition of the heavy sector in a KKLT scenario depends on the details of the compactification, but we expect it to include the complex structure moduli and some heavy Kähler moduli. In that case the light sector would comprise the remaining Kähler moduli, the visible matter fields and the hidden sector where supersymmetry is spontaneously broken.

In this type of  $F$ -term uplifting mechanisms the couplings between the light fields  $L$  and heavy fields  $H$  are characterised by the separability of the Kähler invariant function of the total theory,

$$G(H, \bar{H}, L, \bar{L}) = G^{(h)}(H, \bar{H}) + G^{(l)}(L, \bar{L}) ,$$

which can be expressed in terms of the Kähler potential and the superpotential as

$$\begin{aligned} K(H, \bar{H}, L, \bar{L}) &= K^{(h)}(H, \bar{H}) + K^{(l)}(L, \bar{L}) , \\ W(H, L) &= W^{(h)}(H) W^{(l)}(L) . \end{aligned}$$

This ansatz is approximately satisfied by the couplings between the frozen complex structure moduli and the Kähler moduli in large volume scenarios (Conlon et al., 2005, 2007, Conlon, 2008). In these models it ensures the consistency of including the non-perturbative effects with the supersymmetric integration of the complex structure moduli.<sup>3</sup>

The key property of this type of coupling is that the heavy fields remain at a supersymmetric configuration after coupling them to the light sector, even when supersymmetry is broken by the light fields. In view of the direct couplings in the superpotential it might appear that the two sectors are strongly interacting, and thus that the supersymmetry breaking is likely to spoil the stabilisation of the heavy moduli.

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<sup>3</sup>We thank Joe Conlon for a discussion on this point.

However, a more careful analysis reveals that the two sectors are essentially decoupled. For instance, the mass matrix of field fluctuations around the uplifted vacuum is block diagonal in the light and heavy directions. This implies that the stability analysis can be done independently for the light and heavy sectors. In this chapter we have focused on the study of the stability of the heavy moduli that are truncated. Our results completely confirm and generalise those of the toy model considered in Achúcarro and Sousa (2008), where the heavy sector consisted of a single modulus. More precisely, there is always a basis such that the mass matrix and the Kähler metric can be diagonalised simultaneously. This allows expressing the stability requirement of having a positive definite mass matrix as a constraint on the curvature of the Kähler function at the uplifted vacuum (eq. 3.47). We found that the stability diagram obtained for the toy model holds separately for each eigenvalue of the mass matrix of the uplifted scalar potential, figure 3.1. In particular, our results show that if the heavy fields are fixed at a minimum of the Kähler function the configuration remains stable for any final value of the cosmological constant. However, if the heavy fields are fixed at a saddle point of the Kähler function – the Kähler function cannot have maxima – the configuration always becomes unstable for large enough values of the cosmological constant.

This analysis complements that of Covi et al. (2008b), who formulated a necessary (and in most practical situations sufficient) condition for the existence of (meta)-stable de Sitter vacua, following earlier work by Gomez-Reino and Scrucca (2006b,a, 2007). The constraint restricts the Kähler geometry of the non-linear sigma model associated to the chiral multiplets. Expressed in terms of the metric  $G_{I\bar{J}}$  and the Riemann tensor  $R_{I\bar{J}M\bar{N}}$  of the Kähler manifold it reads

$$\sigma \equiv \left[ \frac{1}{3} \left( G_{I\bar{J}} G_{M\bar{N}} + G_{I\bar{N}} G_{M\bar{J}} \right) - R_{I\bar{J}M\bar{N}} \right] G^I G^{\bar{J}} G^M G^{\bar{N}} > 0. \quad (3.60)$$

This condition, they point out, is e.g. not satisfied by moduli with no-scale Kähler functions of the form  $K = -3 \log(\xi + \bar{\xi})$ , or more generally  $K = -\sum_I n_I \log(\xi^I + \bar{\xi}^{\bar{I}})$ ,  $\sum_I n_I = 3$ . Clearly, the constraint (eq. 3.60) is only sensitive to the geometry of the Kähler manifold along the direction of the goldstino vector  $G_I$ , and therefore it can say nothing about the perturbative stability of moduli with zero  $F$ -terms,  $G_I = 0$ . In particular, it cannot be used to restrict the interactions of those fields that are supersymmetrically decoupled – in the sense of the previous chapter – from the sector that breaks supersymmetry. Our work provides necessary and sufficient conditions for the perturbative stability of these  $G^I = 0$  fields in a particular class of models where they are supersymmetrically decoupled.

Finally, we have also confirmed that the one-to-one correspondence found in

### **Chapter 3: $F$ -term uplifting and the supersymmetric integration of heavy scalars**

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Achúcarro and Sousa (2008) between local minima of the Kähler invariant function  $G$  and (stable) AdS supersymmetric vacua that are local maxima of the scalar potential is completely general. These supersymmetric vacua satisfy the Breitenlohner-Freedman bound and are therefore stable. Our results imply that supersymmetric AdS maxima remain perturbatively stable when supersymmetry is broken by a supersymmetrically decoupled sector satisfying (eq. 3.4). Moreover, we have been able to prove that even in more general scenarios where the truncated heavy moduli do not satisfy (eq. 3.4), the supersymmetric AdS maxima are always stable for large values of the cosmological constant. To our knowledge, the uplifting of (AdS) supersymmetric local *maxima* of the scalar potential has not been considered before and constitutes a new class of stable de Sitter vacua and inflationary troughs whose phenomenology has to be explored.

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## CHAPTER 4

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# Heavy physics in the Cosmic Microwave Background

### 4.1 Introduction

One particularly compelling possibility which will be discussed in the next two chapters is the possibility of features in the spectrum of perturbations that are generated by heavy – relative to the scale of inflation – degrees of freedom which do not necessarily decouple from the dynamics of the inflaton. Although the effects of massive degrees of freedom on the density perturbations are known to quickly dissipate during inflation, there are evidently still a number of contexts where features in the primordial spectrum due to heavy physics can survive. It is well understood, for example, that departures from a Bunch-Davies vacuum as the initial condition for the scalar fluctuations will result in oscillatory features in the power spectrum (see for example Martin and Brandenberger, 2001, Kempf and Niemeyer, 2001, Easther et al., 2001, Danielsson, 2002, Kaloper et al., 2002 and Schalm et al., 2004. For a recent review, see Jackson and Schalm, 2010). Other contexts in which features are generated in the power spectrum involve particle production during brief intervals – much smaller than an  $e$ -fold – as the universe inflates. Examples of this include those situations where a massive field coupled to the inflaton suddenly becomes massless at a specific point in field space (Chung et al., 2000, Elgaroy et al., 2003, Mathews et al., 2004, Romano and Sasaki, 2008, Barnaby and Huang, 2009). Here it is the transfer of energy out of the inflaton field and the subsequent backscatter of its fluctuations

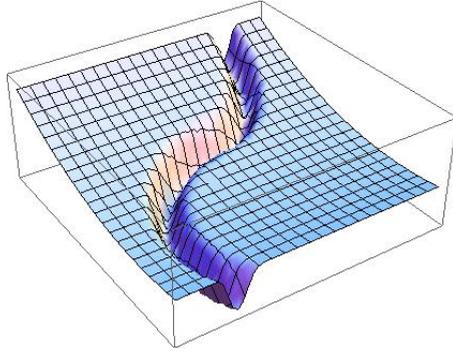
off the condensate of created quanta that can result in features in the power spectrum, as well as in its higher moments (Barnaby and Huang, 2009, Barnaby, 2010). Yet another context where such features have been shown to arise is in chain inflation, where instead of slowly rolling down a smooth continuous potential, the inflaton field gradually tunnels a succession of many vacua (Chialva and Danielsson, 2009).

The purpose of these chapters is to understand, in the context of inflation embedded in a multi-scalar field theory, the general conditions under which features in the power spectrum are generated (see work by Langlois and Renaux-Petel 2008, Peterson and Tegmark 2010, Cremonini et al. 2010a for other recent discussions on this). For this we consider models of inflation where all of the scalar fields remain heavy except for one, the inflaton, which rolls slowly in some multi-dimensional potential. An effective field theory analysis tells us that in such scenarios, inflation should proceed in exactly the same way as in the single-field case, with subleading corrections suppressed by the masses of the heavy scalar fields, see for example Weinberg (2008). In this framework it is easy to take for granted that a simple truncation of any available heavy degrees of freedom is the same as having integrated them out. However, it can certainly be the case that the adiabatic approximation is no longer valid at some point along the inflaton trajectory, e.g. due to a “sudden” turn that mixes heavy and light directions, and higher derivative operators in the effective theory are no longer negligible even as inflation continues uninterrupted.

In various models of inflation in supergravity and string theory, the inflaton is embedded in a non-linear sigma model with typical field manifold curvatures of the string or Planck scale (Gomez-Reino and Scrucca, 2006a, Covi et al., 2008b,a). In this type of scenario the inflaton traverses a curvilinear trajectory generating derivative interactions between the adiabatic and non-adiabatic modes<sup>1</sup> (Gordon et al., 2001, Groot Nibbelink and van Tent, 2000, 2002). In this context, it is straightforward to appreciate heuristically that a sudden enough turn can excite modes normal to the trajectory and non-trivially modify the evolution of the adiabatic mode. We will see that the net effect of this trajectory will translate into damped oscillatory features superimposed on the power spectrum – the transients after a sudden transfer of energy between the excited heavy modes and the much lighter inflaton mode, and the subsequent rescattering of its perturbations off the condensate of heavy quanta that redshift

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<sup>1</sup>Here, by adiabatic mode we refer to the mode which fluctuates along the inflationary trajectory whereas non-adiabatic modes correspond to those whose fluctuations remain orthogonal to the trajectory. We will also frequently denote them as curvature and isocurvature modes in this chapter.



**Figure 4.1:** A generic example of a potential where turns happens while one of the fields remain much heavier than the other.

in short time.<sup>2</sup>

A typical potential exhibiting such a curved trajectory is depicted in figure 4.1. It can be appreciated that there is always a heavy direction transverse to the loci of minima determining the inflaton trajectory. We should emphasise however that the focus of this work is more general and that a curved trajectory in field space is not exclusively due to the shape of the potential, but also depends on the particular sigma model metric defining the scalar field manifold: on a particular curve the two can be transformed into each other by suitable field redefinitions. With this perspective, we will show that curved trajectories appear in any situation where a mismatch exists between the span of geodesics of the scalar field manifold and the actual inflationary trajectory enforced by the scalar potential through the equations of motion. The previously described situation is in fact generic of realisations of inflation in the context of string compactifications, where a large number of scalar fields are expected to remain massive but with their vacuum expectation values depending on the field value of the background inflaton (Blanco-Pillado et al., 2004, Lalak et al., 2007c, Conlon and Quevedo, 2006, Blanco-Pillado et al., 2006b, Simon et al., 2006, Bond et al., 2007, de Carlos et al., 2007, Lalak et al., 2007b, Grimm, 2008, Linde and Westphal, 2008).

Limits of certain cases we wish to study in this chapter have been explored recently in seemingly different, but related contexts. In Chen and Wang (2010a,b), for

<sup>2</sup>We also note the investigations of Tye et al. (2009) and Tye and Xu (2010), where inflation in a putative string landscape is modelled using a random potential. Here, the background inflaton effectively executes a random walk, resulting in features at all scales in the power spectrum.

instance, the effects on density perturbations due to a circular turn with constant curvature in field space was explored within a two-field model. There, it was concluded that such a turn could render non-Gaussian features in the bispectrum but would not generate features in the power spectrum. In another recent publication by Tolley and Wyman (2010), the effects of a sigma model with non-canonical kinetic terms motivated by string theory were explored within inflationary models where one of the fields remained very massive. There, an effective theory was derived describing the multi-field dynamics, characterised by having a speed of sound for the fluctuations smaller than unity (and therefore indicating the possible departure from Gaussianity of the CMB temperature anisotropies). In the framework we are about to discuss, both examples are just different faces of the same coin: while a non-canonical sigma model metric can always be made locally flat along a given trajectory this generally generates contributions to the potential with a curved locus of minima. On the other hand, it is also possible to find a field redefinition which makes the loci of flat directions of the potential look straight at the cost of introducing a non-canonical metric.

We have organised this chapter in the following way. In section 4.2 we present the general setup and the notations used throughout this and the next chapter, extending the work of Groot Nibbelink and van Tent (2000, 2002). There, we will emphasise the need for using a geometric perspective to describe the evolution of the homogeneous background. Then, in section 4.3 we proceed to examine the perturbations of the fields around a time dependent background and consider their quantisation and provide general formulae for the power spectrum. Our formalism allows us to consider situations beyond the regime of applicability of existing methods, such as trajectories with fast, sudden turns (regardless of whether the sigma model metric is canonical or non-canonical), and any other situations in which the masses in the orthogonal direction are changing relatively fast along the trajectory while still remaining much heavier than  $H^2$ . Then, in section 4.4 we will first derive the dynamics of turning fields in the Minkowski limit. In this section we will discuss the useful two-field model and its constant turn limit. Additionally, in the limit of large hierarchy we will show that the dynamics can be described by an effective single-field theory for the light field, with a reduced speed of sound. This chapter will be concluded by a discussion on the validity of truncating non-decoupled sectors, as discussed also in chapter 2. In the next chapter we will apply this framework for calculations of features in the inflationary power spectrum.

## 4.2 Basic considerations

Let us start our study by recalling some of the basic aspects of multi-field inflation and by introducing the notations and conventions that will be used throughout this work. Our starting point is to assume the following effective four dimensional action consisting of gravity and a set of  $\mathcal{N}$  scalar fields  $\phi^a$ :

$$S = \int \sqrt{-g} d^4x \left[ \frac{M_{\text{Pl}}^2}{2} R - \frac{1}{2} \gamma_{ab} g^{\mu\nu} \partial_\mu \phi^a \partial_\nu \phi^b - V(\phi) \right]. \quad (4.1)$$

Here  $R$  denotes the Ricci scalar constructed out of the spacetime metric  $g_{\mu\nu}$  with determinant  $g$ . Additionally,  $\phi^a$  ( $a = 1, \dots, \mathcal{N}$ ) denotes a set of scalar fields spanning a scalar manifold  $\mathcal{M}$  of dimension  $\mathcal{N}$ , equipped with a scalar metric  $\gamma_{ab}$ . The scalar fields may be thought of as coordinates on  $\mathcal{M}$  with Christoffel symbols given by

$$\Gamma_{bc}^a = \frac{1}{2} \gamma^{ad} (\partial_b \gamma_{dc} + \partial_c \gamma_{bd} - \partial_d \gamma_{bc}), \quad (4.2)$$

where  $\partial_a$  are partial derivatives with respect to the scalar fields  $\phi^a$ . In terms of these, the Riemann tensor associated with  $\mathcal{M}$  is given by

$$\mathbb{R}^a{}_{bcd} = \partial_c \Gamma_{bd}^a - \partial_d \Gamma_{bc}^a + \Gamma_{ce}^a \Gamma_{db}^e - \Gamma_{de}^a \Gamma_{cb}^e. \quad (4.3)$$

It is also possible to define the Ricci tensor as  $\mathbb{R}_{ab} = \mathbb{R}^c{}_{acb}$  and the Ricci scalar  $\mathbb{R} = \gamma^{ab} \mathbb{R}_{ab}$ . We shall be careful to distinguish geometrical quantities related to the four dimensional spacetime and the  $\mathcal{N}$ -dimensional abstract manifold  $\mathcal{M}$ . We should keep in mind that, typically, there will be an energy scale  $\Lambda_{\mathcal{M}}$  associated to the curvature of  $\mathcal{M}$ , and hence, fixing the typical mass scale of the Ricci scalar as  $\mathbb{R} \sim \Lambda_{\mathcal{M}}^{-2}$ . In many concrete situations, such as the modular sector of string compactifications, the scale  $\Lambda_{\mathcal{M}}$  corresponds to the Planck mass  $M_{\text{Pl}}$ . The equations of motion for the scalar fields are given by

$$\square \phi^a + \Gamma_{bc}^a g^{\mu\nu} \partial_\mu \phi^b \partial_\nu \phi^c = V^a, \quad (4.4)$$

where  $V^a \equiv \gamma^{ab} \partial_b V$ . In what follows we discuss in detail the homogeneous solutions  $\phi^a = \phi_0^a(t)$  to these equations where the scalar fields depend only on time. In this section and section 4.3 we closely follow the formalism of Groot Nibbelink and van Tent (2000, 2002), extended to allow for the possibility of sharp turns before and around horizon exit.

### 4.2.1 Background solution

We look for background solutions by assuming that all the scalar fields are time dependent  $\phi^a = \phi_0^a(t)$ , and that spacetime consists of a flat Friedmann-Lemaître-Robertson-Walker (FLRW) geometry (eq. 1.1) of the form

$$ds^2 = -dt^2 + a^2(t)\delta_{ij}dx^i dx^j . \quad (4.5)$$

Later on we will also work in conformal time  $\tau$ , defined through the relation  $dt = a d\tau$ . In this background, the equation of motion (eq. 4.4) describing the evolution of the scalar fields is given by

$$\frac{D}{dt}\dot{\phi}_0^a + 3H\dot{\phi}_0^a + V^a = 0 , \quad (4.6)$$

where  $H = \dot{a}/a$  is the Hubble parameter characterising the expansion rate of spatial slices, and where we have also introduced the convenient notation  $DX^a = dX^a + \Gamma_{bc}^a X^b d\phi_0^c$ . On the other hand, the Friedmann equation describing the evolution of the scale factor (eq. 1.2) in terms of the scalar field energy density is given by

$$H^2 = \frac{1}{3M_{\text{Pl}}^2} \left( \frac{1}{2}\dot{\phi}_0^2 + V \right) , \quad (4.7)$$

where  $\dot{\phi}_0^2 \equiv \gamma_{ab}\dot{\phi}_0^a\dot{\phi}_0^b$ . We thus see that  $\dot{\phi}_0$  corresponds to the rate of change of the scalar field vacuum expectation value along the trajectory followed by the background fields, we will assume that  $\dot{\phi}_0 > 0$  everywhere. It is also convenient to recall the following equation describing the variation of  $H$ ,

$$\dot{H} = -\frac{\dot{\phi}_0^2}{2M_{\text{Pl}}^2} , \quad (4.8)$$

which may be deduced by combining (eq. 4.6) and (eq. 4.7). By specifying the metric  $\gamma_{ab}$  and the scalar potential  $V$ , these equations can be solved to obtain the curved trajectory in  $\mathcal{M}$  followed by the scalar fields. To discuss several features of this trajectory without explicitly solving the previous equations, it is useful to define unit vectors  $T^a$  and  $N^a$  distinguishing tangent and normal directions to the trajectory respectively, in such a way that  $T^a N_a = 0$ . These are defined as

$$\begin{aligned} T^a &\equiv \frac{\dot{\phi}_0^a}{\dot{\phi}_0} , \\ N^a &\equiv s_N(t) \left( \gamma_{bc} \frac{DT^b}{dt} \frac{DT^c}{dt} \right)^{-1/2} \frac{DT^a}{dt} , \end{aligned} \quad (4.9)$$

where  $s_N(t) = \pm 1$ , denoting the orientation of  $N^a$  with respect to the vector  $DT^a/dt$ . That is, if  $s_N(t) = +1$  then  $N^a$  is pointing in the same direction as  $DT^a/dt$ , whereas if  $s_N(t) = -1$  then  $N^a$  is pointing in the opposite direction. Due to the presence of the square root, it is clear that  $N^a$  is only well defined at intervals where  $DT^a/dt \neq 0$ . However, since  $DT^a/dt$  may become zero at finite values of  $t$ , we allow  $s_N(t)$  to flip signs each time this happens in such a way that both  $N^a$  and  $DT^a/dt$  remain a continuous function of  $t$ . This implies that the sign of  $s_N$  may be chosen conventionally at some initial time  $t_i$ , but from then on it is subject to the equations of motion respected by the background.<sup>3</sup> In the particular case where  $\mathcal{M}$  is two dimensional, the presence of  $s_N(t)$  in (eq. 4.9) is sufficient for  $N^a$  to have a fixed orientation with respect to  $T^a$  (either left-handed or right-handed). This will become particularly useful when we examine two dimensional models in sections 4.4 and 5.2.

Observe that the tangent vector  $T^a$  offers an alternative way of defining the total time derivative  $D/dt$  along the trajectory followed by the scalar fields,

$$\frac{D}{dt} \equiv \dot{\phi}_0 T^a \nabla_a = \dot{\phi}_0 \nabla_{\dot{\phi}} . \quad (4.10)$$

Now, taking a total time derivative of  $T^a$ , we may use the equation of motion (eq. 4.6) to write

$$\frac{DT^a}{dt} = -\frac{\ddot{\phi}_0}{\dot{\phi}_0} T^a - \frac{1}{\dot{\phi}_0} (3H\dot{\phi}_0^a + V^a) . \quad (4.11)$$

Then, by projecting this equation along the two orthogonal directions  $T^a$  and  $N^a$ , we obtain the following two independent equations

$$\ddot{\phi}_0 + 3H\dot{\phi}_0 + V_\phi = 0 , \quad (4.12)$$

$$\frac{DT^a}{dt} = -\frac{V_N}{\dot{\phi}_0} N^a , \quad (4.13)$$

where we have defined  $V_\phi \equiv T^a V_a$  and  $V_N \equiv N^a V_a$  to be the projections of  $V_a = \partial_a V$  along the tangent and normal directions respectively. It is not difficult to verify that  $V_a$  lies entirely along a space spanned by  $T^a$  and  $N^a$ . That is, we are allowed to write  $V_a \equiv V_\phi T_a + V_N N_a$ . To anticipate the study of inflation within the present setup, it is

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<sup>3</sup>We are assuming here that the background solutions  $\phi^a = \phi_0^a(t)$  are analytic functions of time and that  $\dot{\phi}_0$  is nonvanishing. Under these conditions this procedure can always be performed.

useful to define the dimensionless quantities<sup>4</sup>

$$\epsilon \equiv -\frac{\dot{H}}{H^2} = \frac{\dot{\phi}_0^2}{2M_{\text{pl}}^2 H^2}, \quad (4.14)$$

$$\eta^a \equiv -\frac{1}{H\dot{\phi}_0} \frac{D\dot{\phi}_0^a}{dt}, \quad (4.15)$$

which are the multi-field equivalents of (eq. 1.5). We will not assume that these parameters are small until much later, where inflation is studied in the slow-roll regime (see section 5.3 in the next chapter). Similarly to the case of  $V_a$ , the vector  $\eta^a$  may be decomposed entirely in terms of  $T^a$  and  $N^a$  as

$$\eta^a = \eta_{\parallel} T^a + \eta_{\perp} N^a, \quad (4.16)$$

$$\eta_{\parallel} \equiv -\frac{\ddot{\phi}_0}{H\dot{\phi}_0}, \quad (4.17)$$

$$\eta_{\perp} \equiv \frac{V_N}{\dot{\phi}_0 H}, \quad (4.18)$$

where we have used (eq. 4.6) to simplify a few expressions. Observe that  $\eta_{\perp}$  is directly related to the rate of change of the tangent unit vector  $T^a$ , since (eq. 4.13) can be written as

$$\frac{DT^a}{dt} = -H\eta_{\perp} N^a. \quad (4.19)$$

Comparison with (eq. 4.9) shows that  $\text{sign}(\eta_{\perp}) = -s_N$ . This is one of our main reasons for having introduced  $s_N(t)$  in (eq. 4.9): it allows us to keep  $\eta_{\perp}$  continuous and avoid some unnecessary difficulties encountered in the definition of isocurvature modes.<sup>5</sup>

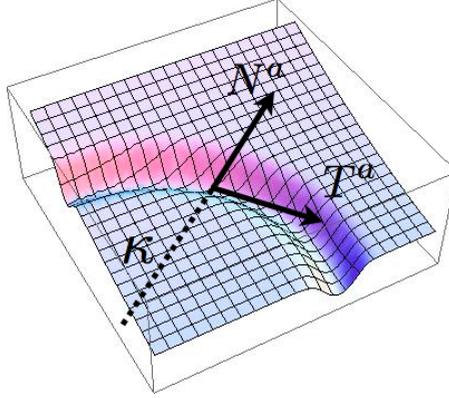
Moving on with this discussion, we can relate  $\eta_{\perp}$  to the radius of curvature  $\kappa$  characterising the bending of the trajectory followed by the scalar fields. To do so, let us recall that given a curve  $\gamma(\phi_0)$  in field space parametrised by  $d\phi_0 = \dot{\phi}_0 dt$ , we may define the radius of curvature  $\kappa$  associated to that curve through the relation

$$\frac{1}{\kappa} = \left( \gamma_{bc} \frac{DT^b}{d\phi_0} \frac{DT^c}{d\phi_0} \right)^{1/2}. \quad (4.20)$$

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<sup>4</sup>Note that our definition of  $\eta^a$  differs from the definition in Groot Nibbelink and van Tent (2000, 2002) by a minus sign.

<sup>5</sup>In Peterson and Tegmark (2010) for instance, a similar parameter  $\eta_{\perp}$  is introduced but with a fixed sign. Partly due to this choice their numerical results cannot handle an overshoot that is occurs when a potential turns from one direction to another.



**Figure 4.2:** The figure shows schematically the relation between the tangent vector  $T^a$ , the normal vector  $N^a$  and the radius of curvature  $\kappa$ .

Here  $\kappa$  stands for the radius of curvature in the scalar manifold  $\mathcal{M}$  spanned by the  $\phi^a$  fields, and therefore it has dimension of mass. Figure 4.2 shows the relation between the pair of vectors  $T^a$ ,  $N^a$  and the radius of curvature  $\kappa$ . Using (eq. 4.10) and comparing the last two equations we find that  $\kappa$  and  $\eta_\perp$  are related as

$$\kappa^{-1} = \frac{H|\eta_\perp|}{\dot{\phi}_0}. \quad (4.21)$$

By definition any autoparallel curve, a curve parallel to a geodesic,  $\gamma(\phi_0)$  in  $\mathcal{M}$  satisfies the relation  $D\dot{\phi}^a/dt \propto \dot{\phi}^a$ , which corresponds to the case  $\kappa^{-1} = 0$ , or alternatively, to the case  $\eta_\perp = 0$ . Thus, we see that the dimensionless parameter  $\eta_\perp$  is a useful quantity that parametrises the bending of the inflationary trajectory with respect to geodesics in  $\mathcal{M}$ . It is interesting to rewrite the previous relation by replacing  $\dot{\phi}_0 = \sqrt{2\epsilon}HM_{\text{Pl}}$  coming from the definition of  $\epsilon$  presented in (eq. 4.14), obtaining

$$|\eta_\perp| = \sqrt{2\epsilon} \frac{M_{\text{Pl}}}{\kappa}. \quad (4.22)$$

Then, if the radius of curvature is such that  $\kappa \ll M_{\text{Pl}}$ , one already sees that  $\eta_\perp^2 \gg 2\epsilon$ . We shall come back to this result later when we study curved trajectories in the slow-roll regime  $\epsilon \ll 1$ . To continue, we may further characterise the variation of  $N^a$  as

$$\frac{DN^a}{dt} = H\eta_\perp T^a + \frac{1}{H\eta_\perp} P^{ab} \nabla_\phi V_b, \quad (4.23)$$

## Chapter 4: Heavy physics in the Cosmic Microwave Background

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where we have defined the projector tensor  $P^{ab} \equiv \gamma^{ab} - T^a T^b - N^a N^b$  along the space orthogonal to the subspace spanned by the unit vectors  $T^a$  and  $N^a$ . That is,  $P_{ab} N^b = 0$  and  $P_{ab} T^b = 0$ .

To obtain (eq. 4.23) we proceed as follows: first, by taking a total time derivative to (eq. 4.6) we obtain

$$\frac{1}{\dot{\phi}_0} \frac{D^2 \dot{\phi}_0^a}{dt^2} = 3H^2(\epsilon T^a + \eta^a) - \nabla_\phi V^a, \quad (4.24)$$

where  $\nabla_\phi \equiv T^a \nabla_a$ . Recalling that  $T^a = \dot{\phi}_0^a / \dot{\phi}_0$ , the previous equation can be re-expressed as

$$\frac{D^2 T^a}{dt^2} = T^a \nabla_\phi V_\phi - \nabla_\phi V^a - \frac{(V_\phi - \dot{\phi}_0) V_N}{\dot{\phi}_0^2} N^a. \quad (4.25)$$

On the other hand, taking a total time derivative to (eq. 4.13) we may obtain yet another expression for the second variation  $D^2 T^a / dt^2$ , given by

$$\frac{D^2 T^a}{dt^2} = \left( \frac{V_N \ddot{\phi}_0}{\dot{\phi}_0^2} - \frac{\dot{V}_N}{\dot{\phi}_0} \right) N^a - \frac{V_N}{\dot{\phi}_0} \frac{DN^a}{dt}. \quad (4.26)$$

Equating the last two expressions and performing some straightforward algebraic manipulations, we finally obtain (eq. 4.23).

To finish this section, let us state some useful relations that will be used throughout the rest of this work. First, by using the definitions for  $\epsilon$  and  $\eta_\parallel$  in (eqs. 4.14 and 4.17), we may rewrite the background equations (eq. 4.7) and (eq. 4.12) respectively as:

$$3 - \epsilon = \frac{V}{M_{\text{Pl}}^2 H^2}, \quad (4.27)$$

$$3 - \eta_\parallel = - \frac{V_\phi}{\dot{\phi}_0 H}. \quad (4.28)$$

With the help of (eq. 4.14) these two relations may be put together to yield:

$$\epsilon = \frac{M_{\text{Pl}}^2}{2} \left( \frac{V_\phi}{V} \right)^2 \left( \frac{3 - \epsilon}{3 - \eta_\parallel} \right)^2. \quad (4.29)$$

Next, by deriving (eq. 4.12) with respect to time and using the definitions for  $\epsilon$  and

$\eta_{\parallel}$ , we deduce<sup>6</sup>

$$3(\epsilon + \eta_{\parallel}) = M_{\text{Pl}}^2 \frac{\nabla_{\phi} V_{\phi}}{V} (3 - \epsilon) + \xi_{\parallel} \eta_{\parallel} , \quad (4.30)$$

$$\xi_{\parallel} \equiv - \frac{1}{H \ddot{\phi}_0} . \quad (4.31)$$

Both (eq. 4.29) and (eq. 4.30) are exact equations linking the evolution of background quantities with the scalar potential  $V$ . It may be already noticed that if  $\epsilon$ ,  $\eta_{\parallel}$  and  $\xi_{\parallel}$  are all much smaller than unity, then we obtain the usual relations for the slow-roll parameters  $\epsilon$  and  $\eta_{\parallel}$  in terms of derivatives of the potential (see eqs. 1.5-1.7):

$$\epsilon \approx \frac{M_{\text{Pl}}^2}{2} \left( \frac{V_{\phi}}{V} \right)^2 , \quad (4.32)$$

$$\epsilon + \eta_{\parallel} \approx M_{\text{Pl}}^2 \frac{\nabla_{\phi} V_{\phi}}{V} . \quad (4.33)$$

We shall come back to these relations later, when we consider the evolution of the background in the slow-roll regime.

### 4.3 Perturbation theory

The notation introduced in the previous section provides a useful tool to analyse perturbations  $\delta\phi^a$  about the background solution  $\phi^a = \phi_0^a(t)$  by decomposing them into parallel and normal components with respect to the inflaton trajectory. In what follows we proceed to study the evolution and quantisation of these perturbations. First, we consider scalar field perturbations by expanding about the background  $\phi^a(t, \mathbf{x}) = \phi_0^a(t) + \delta\phi^a(t, \mathbf{x})$ . It is well known that the equations of motion for the perturbed fields can be cast entirely in terms of the gauge-invariant Sasaki-Mukhanov variables (Sasaki, 1986, Mukhanov, 1988)

$$Q^a \equiv \delta\phi^a + \frac{\dot{\phi}^a}{H} \psi , \quad (4.34)$$

where  $\psi$  is the curvature perturbation of the spatial metric. The equations of motion for these fields are found to be (Sasaki and Stewart, 1996)

$$\frac{D^2 Q^a}{dt^2} + 3H \frac{D Q^a}{dt} - \frac{\nabla^2}{a^2} Q^a + C^a_b Q^b = 0 , \quad (4.35)$$

<sup>6</sup>Note that this definition for  $\xi_{\parallel}$  is different from the definition used in Groot Nibbelink and van Tent (2000, 2002).

## Chapter 4: Heavy physics in the Cosmic Microwave Background

where  $\nabla^2 \equiv \delta^{ij}\partial_i\partial_j$  is the spatial Laplacian and where the tensor  $C^a_b$  is defined as

$$C^a_b \equiv \nabla_b V^a - \dot{\phi}_0^2 \mathbb{R}^a_{\quad cdb} T^c T^d + 2\epsilon \frac{H}{\dot{\phi}_0} (T^a V_b + T_b V^a) + 2\epsilon(3 - \epsilon) H^2 T^a T_b . \quad (4.36)$$

We notice here that  $C_{ab} = \gamma_{ac} C^c_b$  is symmetric. It is convenient to rewrite the set of equations (eq. 4.35) in terms of perturbations orthogonal to each other. With this in mind, we introduce a complete set of vielbeins  $e_a^I = e_a^I(t)$  and work with the following quantities:

$$Q^I(t, \mathbf{x}) \equiv e_a^I(t) Q^a(t, \mathbf{x}) . \quad (4.37)$$

The  $a$ -index labels the abstract scalar manifold  $\mathcal{M}$  whereas the  $I$ -index labels a local orthogonal frame moving along the inflationary trajectory. Recall that vielbeins are defined to satisfy the basic relations  $e_a^I e_b^J \gamma^{ab} = \delta^{IJ}$  and  $e_a^I e_b^J \delta_{IJ} = \gamma_{ab}$ . From these relations one deduces the identities

$$e_a^I \frac{D}{dt} e_J^a = - e_J^a \frac{D}{dt} e_a^I , \quad (4.38)$$

$$e_I^a \frac{D}{dt} e_b^I = - e_b^I \frac{D}{dt} e_I^a , \quad (4.39)$$

from which it is possible to read

$$\dot{Q}^I = e_a^I \frac{D Q^a}{dt} - Y^I_J Q^J , \quad (4.40)$$

$$\ddot{Q}^I = e_a^I \frac{D^2 Q^a}{dt^2} - 2Y^I_J \dot{Q}^J - (Y^I_K Y^K_J + \dot{Y}^I_J) Q^J , \quad (4.41)$$

where the antisymmetric matrix  $Y_{IJ} = -Y_{JI}$  is defined as

$$Y^I_J = e_a^I \frac{D e_J^a}{dt} . \quad (4.42)$$

Before writing down the equations of motion respected by the fields  $Q^I$ , it is useful to notice that the matrix  $Y_{IJ}$  allows us to define a new covariant derivative  $\mathcal{D}/dt$  acting on quantities such as  $Q^I$  labelled with the  $I$ -index in the following way<sup>7</sup>:

$$\frac{\mathcal{D}}{dt} Q^I \equiv \dot{Q}^I + Y^I_J Q^J . \quad (4.43)$$

<sup>7</sup>It may be noticed that we can write  $Y^I_J = (e_a^I \partial_b e_J^a + e_a^I \Gamma_{bc}^a e_J^c) \dot{\phi}_0^b = \omega_b^I{}_J \dot{\phi}_0^b$  where  $\omega_b^I{}_J$  are the usual spin connections for non-coordinate basis, hence justifying the definition of the new covariant derivative of (eq. 4.43).

This definition allows us to rearrange (eq. 4.40) and (eq. 4.41) and simply write

$$\frac{\mathcal{D}Q^I}{dt} = e_a^I \frac{DQ^a}{dt}, \quad (4.44)$$

$$\frac{\mathcal{D}^2 Q^I}{dt^2} = e_a^I \frac{D^2 Q^a}{dt^2}. \quad (4.45)$$

Thus, the equations of motion for the perturbations in the new basis become

$$\frac{\mathcal{D}^2 Q^I}{dt^2} + 3H \frac{\mathcal{D}Q^I}{dt} - \frac{\nabla^2}{a^2} Q^I + C^I{}_J Q^J = 0, \quad (4.46)$$

where  $C_{IJ} \equiv e_{Ia} e_J^b C^a{}_b$ . To deal with this set of equations, it is convenient to take one last step in simplifying them and rewrite them in terms of conformal time  $d\tau = dt/a$ , and a new set of perturbations  $v^I \equiv aQ^I$ . These redefinitions induce a re-scaling of the covariant derivative (eq. 4.43) in the form  $\mathcal{D}/d\tau = a\mathcal{D}/dt$ , from where we are allowed to write

$$\frac{\mathcal{D}v^I}{d\tau} = \frac{dv^I}{d\tau} + Z^I{}_J v^J, \quad (4.47)$$

where  $Z_{IJ} = aY_{IJ}$ . Then, the equations of motion for the  $v^I$ -perturbations are found to be

$$\frac{\mathcal{D}^2 v^I}{d\tau^2} - \nabla^2 v^I + \Omega^I{}_J v^J = 0, \quad (4.48)$$

where  $\Omega_{IJ} = -a^2 H^2 (2 - \epsilon) \delta_{IJ} + a^2 C_{IJ}$  and we have used the definition of  $\epsilon$  to write  $a''/a = a^2 H^2 (2 - \epsilon)$ . For completeness, we notice that the equations of motion (eq. 4.46) may be derived from the action (Groot Nibbelink and van Tent, 2000, 2002)

$$S = \frac{1}{2} \int d\tau d^3x \left[ \sum_I \left( \frac{\mathcal{D}v^I}{d\tau} \right)^2 - \sum_I (\nabla v^I)^2 - \Omega_{IJ} v^I v^J \right], \quad (4.49)$$

which can be alternatively deduced directly from the initial action (eq. 4.1) by considering all of the field redefinitions introduced in the present discussion.

The set of equations (eq. 4.48) contains several non-trivial features. First, notice that the covariant derivative  $\mathcal{D}/d\tau$  implies the existence of non-trivial couplings affecting the kinetic term of each field  $v^I$ . By the same token, under general circumstances the symmetric matrix  $\Omega_{IJ}$  does not remain diagonal at all times. In fact, as we are about to see in the next section, it is possible to choose to write this theory either in a frame where the  $\mathcal{N}$  scalar fields are canonical (and therefore without non-trivial couplings in the kinetic term), or either in a frame where  $\Omega_{IJ}$  remains diagonal, but (in general) not both at the same time.

### 4.3.1 Canonical frame

Observe that by introducing the vielbeins  $e_a^I$  in the previous section, we have not specified any alignment of the moving frame. In fact, given an arbitrary frame, characterised by the set  $e_a^I$ , it is always possible to find a canonical frame where the scalar field perturbations acquire canonical kinetic terms in the action. To find it, let us introduce a new set of fields  $u^I$  defined out of the original fields  $v^I$  in the following way:

$$v^I(\tau, \mathbf{x}) = R^I{}_J(\tau, \tau_i) u^J(\tau, \mathbf{x}), \quad (4.50)$$

where  $R^I{}_J(\tau, \tau_i)$  is an invertible matrix defined to satisfy the first order differential equation

$$\frac{d}{d\tau} R^I{}_J = -Z^I{}_K R^K{}_J, \quad (4.51)$$

with the boundary condition  $R^I{}_J(\tau_i, \tau_i) = \delta^I{}_J$  set at some given initial time  $\tau_i$ . Let us additionally define a new matrix  $S^I{}_J$  to be the inverse of  $R^I{}_J$ , i.e.  $S^I{}_K R^K{}_J = R^I{}_K S^K{}_J = \delta^I{}_J$ . Then,  $S^I{}_J$  satisfies the similar equation

$$\frac{d}{d\tau} S^I{}_J = -Z^J{}_K S^I{}_K, \quad (4.52)$$

where we used the fact that  $Z_{IJ} = -Z_{JI}$ . Since both solutions to (eq. 4.51) and (eq. 4.52) are unique, then the previous equation tells us that  $S_{IJ} = R_{JI}$ , that is,  $S$  corresponds to  $R^T$  the transpose of  $R$ . This means that for a fixed time  $\tau$ ,  $R_{IJ}(\tau, \tau_i)$  is an element of the orthogonal group  $O(N)$ , the group of matrices  $R$  satisfying  $RR^T = \mathbb{1}$ . The solution to (eq. 4.51) is well known, and may be symbolically written as

$$R(\tau, \tau_i) = \mathbb{1} + \sum_{n=1}^{\infty} \frac{(-1)^n}{n!} \int_{\tau_i}^{\tau} \mathcal{T} [Z(\tau_1) \cdots Z(\tau_n)] d^n \tau = \mathcal{T} \exp \left[ - \int_{\tau_i}^{\tau} d\tau Z(\tau) \right], \quad (4.53)$$

where  $\mathcal{T}$  stands for the usual time ordering symbol, that is  $\mathcal{T} [Z(\tau_1) Z(\tau_2) \cdots Z(\tau_n)]$  corresponds to the product of  $n$  matrices  $Z(\tau_i)$  for which  $\tau_1 \geq \tau_2 \geq \cdots \geq \tau_n$ . Coming back to the  $u^I$ -fields, it is possible to see now that, by virtue of (eq. 4.51) one has

$$\frac{\mathcal{D}v^I}{d\tau} = R^I{}_J \frac{du^J}{d\tau}, \quad (4.54)$$

$$\frac{\mathcal{D}^2 v^I}{d\tau^2} = R^I{}_J \frac{d^2 u^J}{d\tau^2}. \quad (4.55)$$

Inserting these relations back into the equation of motion (eq. 4.48) we obtain the following equation of motion for the  $u^I$ -fields:

$$\frac{d^2 u^I}{d\tau^2} - \nabla^2 u^I + \left[ R^T(\tau) \Omega R(\tau) \right]^I{}_J u^J = 0. \quad (4.56)$$

Additionally, it is possible to show that the action (eq. 4.49) is now given by

$$S = \frac{1}{2} \int d\tau d^3x \left\{ \sum_I \left( \frac{du^I}{d\tau} \right)^2 - (\nabla u^I)^2 - [R^T(\tau)\Omega R(\tau)]_{JJ} u^I u^I \right\}. \quad (4.57)$$

Thus, we see that the fields  $u^I$  correspond to the canonical fields in the usual sense. This result shows, just as we have stated, that it is always possible to find a frame where the perturbations become canonical, but at the cost of having a mass matrix  $[R^T(\tau)\Omega R(\tau)]_{JJ}$  with non-diagonal entries which are changing continuously in time. Another way to put it is that, while both  $R^T(\tau)\Omega R(\tau)$  and  $\Omega$  share the same eigenvalues, as long as  $R(\tau)$  varies in time, their associated eigenvectors will not remain aligned. To finish, let us notice that by construction, at the initial time  $\tau_i$ , the canonical fields  $u^I$  and the original fields  $v^I$  coincide  $u^I(\tau_i) = v^I(\tau_i)$ . However, it is always possible to redefine a new set of canonical fields by performing an orthogonal transformation of the fields.

### 4.3.2 Quantisation and initial conditions

Having the canonical frame at hand, we may now quantise the system in the standard way. Starting from the action (eq. 4.57) it is possible to see that the canonical coordinate fields are given by  $u^I$  whereas the canonical momentum is given by  $\Pi_u^I = du^I/d\tau$ . To quantise the system, we demand this pair to satisfy the commutation relation

$$[u^I(\tau, \mathbf{x}), \Pi_u^J(\tau, \mathbf{y})] = i\delta^{IJ}\delta^{(3)}(\mathbf{x} - \mathbf{y}), \quad (4.58)$$

otherwise zero. With the help of the  $R$  transformation introduced in (eq. 4.50) we can rewrite this commutation relation to be valid in an arbitrary moving frame. More precisely, we observe here that we are allowed to define a new pair of fields  $v^I$  and  $\Pi_v^I$  given by

$$v^I = R^I{}_J u^J, \quad (4.59)$$

$$\Pi_v^I \equiv \frac{\mathcal{D}}{d\tau} v^I = R^I{}_J(\tau, \tau_i) \Pi_u^J. \quad (4.60)$$

From (eq. 4.58), this new pair is found to satisfy the similar commutation relations

$$[v^I(\tau, \mathbf{x}), \Pi_v^J(\tau, \mathbf{y})] = i\delta^{IJ}\delta^{(3)}(\mathbf{x} - \mathbf{y}). \quad (4.61)$$

Following convention, it is now possible to obtain an explicit expression for  $v^I(\mathbf{x}, \tau)$  in terms of creation and annihilation operators.<sup>8</sup> For this, let us write  $v^I(\mathbf{x}, \tau)$  as a sum of Fourier modes:

$$\begin{aligned} v^I(\tau, \mathbf{x}) &= \int \frac{d^3k}{(2\pi)^{3/2}} e^{i\mathbf{k} \cdot \mathbf{x}} v^I(\mathbf{k}, \tau) \\ &= \int \frac{d^3k}{(2\pi)^{3/2}} e^{i\mathbf{k} \cdot \mathbf{x}} \sum_{\alpha} \left[ v_{\alpha}^I(k, \tau) a_{\alpha}(\mathbf{k}) + v_{\alpha}^{I*}(k, \tau) a_{\alpha}^{\dagger}(-\mathbf{k}) \right]. \end{aligned} \quad (4.62)$$

In writing the previous expression we have anticipated the need of expressing the fields  $v^I(\tau, \mathbf{x})$  as a linear combination of  $\mathcal{N}$  time-independent creation and annihilation operators  $a_{\alpha}^{\dagger}(\mathbf{k})$  and  $a_{\alpha}(\mathbf{k})$  respectively, with  $\alpha = 1, \dots, \mathcal{N}$ . These operators are required to satisfy the usual relations

$$\left[ a_{\alpha}(\mathbf{k}), a_{\beta}^{\dagger}(\mathbf{q}) \right] = \delta_{\alpha\beta} \delta^{(3)}(\mathbf{k} - \mathbf{q}), \quad (4.63)$$

otherwise zero. This set of operators defines the vacuum  $|0\rangle$  of the theory by their action  $a_{\alpha}(\mathbf{k})|0\rangle = 0$ . Since the operators  $a_{\alpha}^{\dagger}(\mathbf{k})$  and  $a_{\alpha}(\mathbf{k})$ , for different values of  $\alpha$ , are taken to be linearly independent, then the time-dependent coefficients  $v_{\alpha}^I(k, \tau)$  appearing in front of them in (eq. 4.62) must satisfy the equation of motion<sup>9</sup>

$$\frac{\mathcal{D}^2}{dt^2} v_{\alpha}^I(k, \tau) + k^2 v_{\alpha}^I(k, \tau) + \Omega^I_J v_{\alpha}^J(k, \tau) = 0. \quad (4.64)$$

Observe that there must exist  $\mathcal{N}$  independent solutions  $v_{\alpha}^I(k, \tau)$  to this equation (see appendix A for a detailed discussion on the  $v_{\alpha}^I(k, \tau)$ -functions).

Of course, a critical issue here is to set the correct initial conditions for the mode amplitudes  $v_{\alpha}^I(k, \tau)$  in such a way that the commutation relations (eq. 4.61) are respected at all times. As a first step towards determining these initial conditions we notice that at a given initial time  $\tau = \tau_i$  we may choose each mode  $v_{\alpha}^I(k, \tau)$  to satisfy the following general initial conditions:

$$v_{\alpha}^I(k, \tau_i) = e_{\alpha}^I v_{\alpha}(k), \quad (4.65)$$

$$\frac{\mathcal{D}v_{\alpha}^I}{dt}(k, \tau_i) = e_{\alpha}^I \pi_{\alpha}(k), \quad (4.66)$$

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<sup>8</sup>From this point on, we continue working with the more general  $v^I$ -fields instead of the canonical  $u^I$ -fields. Nevertheless, we emphasise that the  $u^I$ -fields allowed us to find the correct quantisation prescription for the  $v^I$ -fields.

<sup>9</sup>It is crucial to appreciate that the Greek indices  $\alpha$  label scalar quantum modes and not directions in field space, as capital Latin indices do. Different  $\alpha$ -modes may contribute to the same fluctuation along a given direction  $I$ . The quantities linking these two different abstract spaces are the mode functions  $v_{\alpha}^I(k, \tau)$  whose time evolution is dictated by (eq. 4.64). A similar scheme to quantise a coupled multi-scalar field system may be found in Nilles et al. (2001).

where  $e_\alpha^I$  is a complete set of unit vectors satisfying  $\delta_{IJ}e_\alpha^I e_\beta^J = \delta_{\alpha\beta}$  and  $\delta^{\alpha\beta}e_\alpha^I e_\beta^J = \delta^{IJ}$ , which should not be confused with the vielbeins defined in (eq. 4.37), and  $v_\alpha(k)$  and  $\pi_\alpha(k)$  are factors defining the amplitude of the initial conditions. In order for the commutation relations to be fulfilled, these initial conditions must satisfy

$$v_\alpha(k)\pi_\alpha^*(k) - v_\alpha^*(k)\pi_\alpha(k) = i, \quad (4.67)$$

which are the analogous relations to the Wronskian condition in single-field slow-roll inflation. Since the operator  $\mathcal{D}/d\tau = d/d\tau + Z$  mixes different directions in the  $v^I$ -field space and since in general the time-dependent matrix  $\Omega_{IJ}$  is non-diagonal, then the mode solutions  $v_\alpha^I(k, \tau)$  satisfying the initial conditions (eq. 4.67) will not remain pointing in the same direction (nor will they remain orthogonal) at an arbitrary time  $\tau \neq \tau_i$ . In Appendix A we show that the commutation relations of (eq. 4.61) are consistent with the evolution of the  $v_\alpha^I(\tau, k)$  dictated by the set of equations of motion (eq. 4.64).

In the previous expressions the set of unit vectors  $e_\alpha^I$  are arbitrary. Moreover, the amplitudes  $v_\alpha(k)$  and  $\pi_\alpha(k)$  entering (eq. 4.67) are in general not uniquely determined, as there is a family of solutions parametrised by the relative phase between  $v_\alpha(k)$  and  $\pi_\alpha(k)$ . Indeed, notice that without loss of generality we may write

$$\pi_\alpha(k) = \frac{e^{-i\theta_\alpha(k)}}{2v_\alpha^*(k) \sin \theta_\alpha(k)}, \quad (4.68)$$

where  $\theta_\alpha(k)$  is a set of real phases relating both amplitudes. Any value of  $\theta_\alpha(k)$  will satisfy the commutation relations (eq. 4.61), and therefore they specify different choices for the vacuum state  $|0\rangle$ . Although in general it is not possible to decide among all the possible values for  $\theta_\alpha(k)$ , fortunately, in the context of inflationary backgrounds  $a \rightarrow 0$  as  $\tau \rightarrow -\infty$  and a particular choice for these phases becomes handy. Indeed, observe that in the formal limit  $a \rightarrow 0$  one has  $Z_{IJ} \rightarrow 0$  and  $\Omega_{IJ} \rightarrow 0$ , which is made explicit by (eq. 4.47) and (eq. 4.48), and the equations of motion (eq. 4.64) become

$$\left( \frac{d^2}{d\tau^2} + k^2 \right) v_\alpha^I(k, \tau) = 0, \quad (\tau \rightarrow -\infty). \quad (4.69)$$

In this limit there is no mixing between different  $\alpha$ -modes and perturbations evolve as if they were in Minkowski background.<sup>10</sup> In this case, we are free to choose  $e_\alpha^I = \delta_\alpha^I$

<sup>10</sup>To be more rigorous, in inflationary backgrounds this limit is obtained for  $k$ -modes such that their wavelength is much smaller than the de Sitter scale  $k^2 \gg a^2 H^2$ .

and the solutions to (eq. 4.69) satisfying the commutation relations (eq. 4.61) may be chosen as

$$v_\alpha^J(k, \tau) = \delta_\alpha^J \frac{1}{\sqrt{2k}} e^{-ik\tau}, \quad (\tau \rightarrow -\infty). \quad (4.70)$$

Thus we see that in the limit  $a \rightarrow 0$  ( $\tau \rightarrow -\infty$ ) we may choose modes in the Bunch-Davies vacuum  $\theta_\alpha = \pi/2$ . We will come back to these conditions in the next chapter, in section 5.4, where we set initial conditions on a finite initial time surface where (eq. 4.70) cannot be exactly imposed.

### 4.3.3 Two-point correlation function

To finish this general discussion on multi-field perturbations, we proceed to define the spectrum for the perturbations  $v^J(\tau, \mathbf{x})$ . The power spectrum, the Fourier transform of the two-point correlation function, is defined in terms of the Fourier modes as

$$\langle 0 | v^J(\mathbf{k}, \tau) v^{J*}(\mathbf{q}, \tau) | 0 \rangle \equiv \delta^{(3)}(\mathbf{k} - \mathbf{q}) \frac{2\pi^2}{k^3} \mathcal{P}_v^{JJ}(k, \tau). \quad (4.71)$$

In terms of the mode amplitudes  $v_\alpha^J(\tau, k)$ , this is found to be

$$\mathcal{P}_v^{JJ}(k, \tau) = \frac{k^3}{2\pi^2} \sum_\alpha v_\alpha^J(\tau, k) v_\alpha^{J*}(k, \tau). \quad (4.72)$$

Since the commutation relations require  $\sum_\alpha [v_\alpha^J(k, \tau) v_\alpha^{J*}(k, \tau) - v_\alpha^J(k, \tau) v_\alpha^{J*}(k, \tau)] = 0$  (see Appendix A) we see that the spectrum  $\mathcal{P}_v^{JJ}$  is real, as it should be. Additionally, the two point correlation functions in coordinate space may be computed out of  $\mathcal{P}_v^{JJ}$  as

$$\langle 0 | v^J(\tau, \mathbf{x}) v^J(\tau, \mathbf{y}) | 0 \rangle = \frac{1}{4\pi} \int \frac{d^3k}{k^3} \mathcal{P}_v^{JJ}(k, \tau) e^{-ik \cdot (\mathbf{x} - \mathbf{y})}. \quad (4.73)$$

We may also define the power spectrum associated to the  $Q^J$  fields instead of the  $v^J$  fields. Recalling that  $Q^J = v^J/a$ , the power spectrum for these fields at a given time  $\tau$  is then given by

$$\mathcal{P}_Q^{JJ}(k, \tau) = \frac{k^3}{2\pi^2 a^2} \sum_\alpha v_\alpha^J(k, \tau) v_\alpha^{J*}(k, \tau). \quad (4.74)$$

This expression will be used to compute the power spectrum of the curvature perturbation produced during inflation. Although, in this section we have chosen to exploit a notation whereby Greek indices  $\alpha$  label quantum modes, notice that this formalism is equivalent to the use of stochastic Gaussian variables, as in Tsujikawa et al. (2003) (see also Lalak et al., 2007b).

## 4.4 Applications in Minkowski space

In this section we will go to the Minkowski limit, meaning that we set the Ricci scalar to zero and replace the action (eq. 4.1) with

$$S = - \int d^4x \left[ \frac{1}{2} \gamma_{ab} \partial^\mu \phi^a \partial_\mu \phi^b + V(\phi) \right], \quad (4.75)$$

Most of the discussion in sections 4.2 and 4.3 remains relevant, where of course all equations are to be understood with  $H = \epsilon = \eta = 0$ . In particular, this means that the equations for the perturbations (eq. 4.48) reduce to

$$\frac{\mathcal{D}^2}{dt^2} v^I - \nabla^2 v^I + C^I{}_J v^J = 0, \quad (4.76)$$

where  $C^I{}_J \equiv e^I{}_a e^b{}_J C^a{}_b$  and

$$C^a{}_b = \nabla_b V^a - \dot{\phi}_0^2 \mathcal{R}^a{}_{cdb} T^c T^d. \quad (4.77)$$

### 4.4.1 Dynamics in the presence of mass hierarchies

The main quantity determining the dynamics of the present system is the scalar potential  $V(\phi)$ . Since we are interested in studying the dynamics of multi-scalar field theories in Minkowski space-time, we will assume that it is positive definite,  $V(\phi) \geq 0$ . From the potential one can define the mass matrix  $M_{ab}^2$  associated to the scalar fluctuations around a given vacuum expectation value  $\langle \phi^a \rangle = \phi_0^a$  as

$$M_{ab}^2(\phi_0) \equiv \nabla_a \nabla_b V|_{\phi=\phi_0}. \quad (4.78)$$

In general, this definition renders a non-diagonal mass matrix, yet it is always possible to find a “local” frame in which it becomes diagonal and with the entries given by the eigenvalues  $m_a^2$ . Now, we take into account the existence of hierarchies among different families of scalar fields, and specifically consider two families, herein referred to as heavy and light fields which are characterised by

$$m_H^2 \gg m_L^2. \quad (4.79)$$

In the particular case where the vacuum expectation value of the scalar fields remains constant  $\dot{\phi}_0^a = 0$ , it is well understood that the heavy fields can be systematically integrated out, providing corrections of  $\mathcal{O}(k^2/m_H^2)$  with  $k$  being the energy scale of interest to the low energy effective Lagrangian describing the remaining light degrees

of freedom (Appelquist and Carazzone, 1975). If however the vacuum expectation value  $\phi_0^a$  is allowed to vary with time, new effects start occurring which can be significant at low energies. We focus on scalar potentials  $V(\phi)$  for which hierarchies are present, and for which the trajectory followed by the scalar fields is such that

$$T^a T^b M_{ab}^2 \sim m_L^2, \quad (4.80)$$

$$N^a N^b M_{ab}^2 \sim m_H^2. \quad (4.81)$$

Since  $M_{ab}^2$  is in general non-diagonal, for consistency we take  $T^a N^b M_{ab}^2$  to be at most of  $\mathcal{O}(m_L m_H)$ . Such trajectories are generic in the following sense: for arbitrary initial conditions, the background field  $\phi_0^a$  typically will start evolving to the minimum of the potential  $V(\phi)$  by first quickly minimising the heavy directions. Then the light modes evolve to their minimum much more slowly.

We will continue the present analysis systematically by splitting the potential into two parts,

$$V(\phi) = V_*(\phi) + \delta V(\phi). \quad (4.82)$$

Here,  $V_*(\phi) \geq 0$  is the zeroth-order positive definite potential characterised by containing exactly flat directions, and  $\delta V(\phi)$  is a correction which breaks this flatness.<sup>11</sup> By construction,  $V_*(\phi)$  contains all the information regarding the heavy directions. Therefore, the mass matrix  $M_{*ab}^2$  obtained out of  $V_*(\phi)$  presents eigenvalues which are either zero or  $\mathcal{O}(m_H^2)$ . Consequently, the light masses appear only after including the correction  $\delta V(\phi)$ . We thus require the second derivatives of  $\delta V(\phi)$  to be at most  $\mathcal{O}(m_L^2)$ . It should be clear that such a splitting is not unique, as it is always possible to redefine both contributions by keeping the property  $M_{*ab}^2 \sim m_H^2$ .

It is clear that the solution to the equation

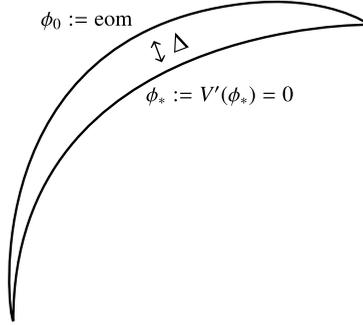
$$V_*^a(\phi) = 0 \quad (4.83)$$

defines a hypersurface  $\mathcal{S}$  in  $\mathcal{M}$ . The dimension of the surface  $\mathcal{S}$  corresponds to the number of flat directions present in  $V(\phi)$ . Let us denote this solution by  $\phi_*^a$ . In appendix B we study in detail the dynamics offered by the zeroth-order theory, in which only the contribution  $V_*(\phi)$  to the potential  $V(\phi)$  is taken into account. For present purposes we quote here a simple result concerning background solutions offered by potentials of this sort:  $\phi_0^a$  and  $\phi_*^a$  are related by

$$\phi_0^a = \phi_*^a + \Delta^a, \quad (4.84)$$

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<sup>11</sup>Such a type of splitting happens in the moduli sector of many low energy string compactifications, where  $V_*$  appear as a consequence of fluxes (Giddings et al., 2002), and  $\delta V(\phi)$  arguably from non-perturbative effects (Kachru et al., 2003a).



**Figure 4.3:** The difference between  $\phi_*$  and  $\phi_0$ .

where  $\Delta^a \simeq N^a \Delta$  parametrises the displacement from  $\mathcal{S}$  of the field trajectory obtained by considering the full potential (eq. 4.82), with  $\Delta$  given by (see figure 4.3)

$$\Delta = \frac{\dot{\phi}_*^2}{m_{*H}^2 \kappa_*} . \quad (4.85)$$

Here,  $\kappa_*$  is the radius of curvature of the projected curve on  $\mathcal{S}$ . We see that the deviation from the surface  $\mathcal{S}$  will be small as long as the dimensionless parameter

$$\frac{\beta}{4} \equiv \frac{\Delta}{\kappa_*} = \frac{\dot{\phi}_*^2}{m_{*H}^2 \kappa_*^2} \quad (4.86)$$

remains small.<sup>12</sup> In what follows, we shall see how this parameter affects the low energy dynamics valid for the light degrees of freedom tangent to  $\mathcal{S}$ . To simplify our analysis, we focus on two-dimensional models. These results can be easily generalised to an arbitrary number of scalar fields.

#### 4.4.2 Two-field models

For theories with two scalar fields we can always choose the set of vielbeins  $e_a^I$  to consist in the following pair:

$$e_1^a = e_T^a = T^a , \quad (4.87)$$

$$e_2^a = e_N^a = N^a . \quad (4.88)$$

<sup>12</sup>As  $\Delta$  is a function of  $\kappa_*$ , this leads to a lower bound on  $\kappa$  for which this approach is valid. For small  $\Delta$  we find that we need to require  $\kappa_* \approx \kappa \gg \dot{\phi}_*^2 / M_{*H}^2$ .

## Chapter 4: Heavy physics in the Cosmic Microwave Background

With this choice we can write  $v^T \equiv T_a \varphi^a$  and  $v^N \equiv N_a \varphi^a$  in a similar fashion as is done in (eq. 4.37), which denote the perturbations parallel and normal to the background trajectory, respectively. In the case where  $\mathcal{M}$  is two-dimensional, these mutually orthogonal vectors are enough to span all of space. Therefore, the two unit vectors satisfy the relations

$$\frac{DT^a}{dt} = -\frac{\dot{\phi}_0}{\kappa} N^a, \quad (4.89)$$

$$\frac{DN^a}{dt} = \frac{\dot{\phi}_0}{\kappa} T^a, \quad (4.90)$$

assuming that  $DT^a/dt$  is nonvanishing and  $s_N(t) < 0$  (this assumes a right turning trajectory, see figure 5.1). In terms of the formalism of the previous sections, these expressions may be written down as  $Z_{TN} = -Z_{NT} = \dot{\phi}_0/\kappa$ . Further, the entries of the symmetric tensor  $C_{IJ} = e_I^a e_J^b C_{ab}$  defined in (eq. 4.77) are given by

$$C_{TT} = T^a T^b \nabla_a V_b, \quad (4.91)$$

$$C_{TN} = T^a N^b \nabla_a V_b, \quad (4.92)$$

$$C_{NN} = N^a N^b \nabla_a V_b + \frac{\dot{\phi}_0^2}{2} \mathcal{R}, \quad (4.93)$$

where  $\mathcal{R} = \gamma^{ab} \mathcal{R}_{acb} = 2\mathcal{R}_{NTN}^T$  is the Ricci scalar.<sup>13</sup> Then, we notice that  $T^a T^b \nabla_a V_b = T^a \nabla_a (T^b V_b) - (T^a \nabla_a T^b) V_b$  and using the fact  $T^a \nabla_a \equiv \nabla_\phi = \dot{\phi}_0^{-1} D/dt$ , we may rewrite

$$C_{TT} = \nabla_\phi V_\phi + \zeta^2, \quad (4.94)$$

$$C_{TN} = \dot{\zeta} - \frac{2V_\phi}{\kappa}, \quad (4.95)$$

where  $\zeta \equiv \dot{\phi}_0/\kappa$  and  $V_\phi \equiv T^a V_a$ . The remaining component  $C_{NN}$  cannot be deduced in this way, as it depends on the second variation of  $V$  away from the trajectory. Inserting the previous expressions back into (eq. 4.48), the set of equations of motion for the pair of perturbations  $v^T$  and  $v^N$  is found to be

$$\ddot{v}^T - \nabla^2 v^T + 2\dot{\zeta} v^N + 2\zeta v^N + \nabla_\phi V_\phi v^T - 2\frac{V_\phi}{\kappa} v^N = 0, \quad (4.96)$$

$$\ddot{v}^N - \nabla^2 v^N - 2\dot{\zeta} v^T + M^2 v^N - 2\frac{V_\phi}{\kappa} v^T = 0, \quad (4.97)$$

<sup>13</sup>Since  $\mathcal{M}$  is two dimensional,  $\mathcal{R}_{NTN}^T = \mathcal{R}/2$  is the only non-vanishing component of the Riemann tensor.

where  $M^2 = C_{NN} - \zeta^2$ . The rotation matrix  $R^I{}_J$  connecting the perturbations  $v^I$  with the canonical counterparts  $u^I$  is easily found to be

$$R^I{}_J = \begin{pmatrix} \cos \theta(t) & -\sin \theta(t) \\ \sin \theta(t) & \cos \theta(t) \end{pmatrix}, \quad (4.98)$$

$$\theta(t) = \int_t dt' \zeta(t'). \quad (4.99)$$

The convenience of staying in the frame where  $e^a_T = T^a$  and  $e^a_N = N^a$  is that the matrix  $C_{IJ}$  has elements with a well defined physical meaning.

### 4.4.3 Constant radius of curvature

To gain some insight into the dynamics behind these equations, let us consider the particular case where  $V_\phi = T^a V_a = 0$  and  $\nabla_\phi V_\phi = 0$ . This is the situation in which the background solution consists of a trajectory in field space crossing an exactly flat valley within the landscape. As  $V_\phi = 0$  requires  $\dot{\phi}_0 = 0$ , we see that  $\phi_0$  becomes a constant of motion. Additionally, let us assume that the radius of curvature  $\kappa$  remains constant, and that the mass matrix  $M^2 = C_{NN} - \zeta^2$  is also constant.<sup>14</sup> Under these conditions  $\zeta$  is a constant and one has  $C_{TT} = \zeta^2$  and  $C_{TN} = 0$ . Then, the equations of motion for the perturbations become

$$\ddot{v}^T - \nabla^2 v^T + 2\zeta \dot{v}^N = 0, \quad (4.100)$$

$$\dot{v}^N - \nabla^2 v^N - 2\zeta \dot{v}^T + M^2 v^N = 0. \quad (4.101)$$

We can solve and quantise these perturbations by following the procedure deduced in section 4.3. First, the mode solutions  $v^I_\alpha(k)$  must satisfy

$$\ddot{v}^T_\alpha + 2\zeta \dot{v}^N_\alpha + k^2 v^T_\alpha = 0, \quad (4.102)$$

$$\dot{v}^N_\alpha - 2\zeta \dot{v}^T_\alpha + (M^2 + k^2) v^N_\alpha = 0. \quad (4.103)$$

To obtain the mode solutions let us try the ansatz

$$v^I_\alpha(k, t) = v^I_\alpha(k) e^{-i\omega_\alpha t}, \quad (4.104)$$

where  $\omega_\alpha \geq 0$  ( $\alpha = 1, 2$ ) corresponds to a set of frequencies to be deduced shortly. Notice that the associated operators  $a^\dagger_\alpha(\mathbf{k})$  and  $a_\alpha(\mathbf{k})$  create and annihilate states char-

<sup>14</sup>Notice that in the particular case where  $\mathcal{M}$  is flat and with a trivial topology, these conditions would correspond to an exact circular curve, such as the one that would happen near the bottom of the ‘Mexican hat’ potential.

acterised by the frequency  $\omega_\alpha$  and momentum  $\mathbf{k}$ . With the former ansatz, the equations of motion take the form

$$(k^2 - \omega_\alpha^2) v_\alpha^T(k) - 2i\omega_\alpha \zeta v_\alpha^N(k) = 0, \quad (4.105)$$

$$(M^2 + k^2 - \omega_\alpha^2) v_\alpha^N(k) + 2i\omega_\alpha \zeta v_\alpha^T(k) = 0. \quad (4.106)$$

Combining them one finds the equation determining the values of  $\omega_\alpha$  as

$$(k^2 - \omega_\alpha^2)(M^2 + k^2 - \omega_\alpha^2) = 4\zeta^2 \omega_\alpha^2. \quad (4.107)$$

The solutions to this equation are

$$\omega_\pm^2 = \frac{1}{2} \left[ (M^2 + 2k^2 + 4\zeta^2) \pm \sqrt{(M^2 + 2k^2 + 4\zeta^2)^2 - 4k^2(M^2 + k^2)^2} \right]. \quad (4.108)$$

On the other hand, the coefficients  $v_\alpha^T(k)$  and  $v_\alpha^N(k)$  must be such that the relations (eq. A.1) and (eq. A.2) are satisfied. After straightforward algebra, it is possible to show that these coefficients are given by

$$|v_-^T(k)|^2 = \frac{(\omega_+^2 - k^2)\omega_-}{2k^2(\omega_+^2 - \omega_-^2)}, \quad |v_-^N(k)|^2 = \frac{(\omega_+^2 - M^2 - k^2)\omega_-}{2(M^2 + k^2)(\omega_+^2 - \omega_-^2)}, \quad (4.109)$$

$$|v_+^T(k)|^2 = \frac{(k^2 - \omega_-^2)\omega_+}{2k^2(\omega_+^2 - \omega_-^2)}, \quad |v_+^N(k)|^2 = \frac{(M^2 + k^2 - \omega_-^2)\omega_+}{2(M^2 + k^2)(\omega_+^2 - \omega_-^2)}. \quad (4.110)$$

If  $\zeta^2 \ll M^2$  we can in fact expand all the relevant quantities in powers of  $\zeta^2$ . One finds, up to leading order in  $\zeta^2/M^2$ ,

$$\omega_- = k \left( 1 - \frac{2\zeta^2}{M^2} \right), \quad (4.111)$$

$$\omega_+ = \sqrt{M^2 + k^2} \left( 1 + \frac{2\zeta^2}{M^2} \right), \quad (4.112)$$

$$|v_-^T(k)|^2 = \frac{1 - 2\zeta^2/M^2}{2k}, \quad (4.113)$$

$$|v_-^N(k)|^2 = \frac{2\zeta^2 k}{M^4}, \quad (4.114)$$

$$|v_+^T(k)|^2 = \frac{2\zeta^2 \sqrt{M^2 + k^2}}{M^4}, \quad (4.115)$$

$$|v_+^N(k)|^2 = \frac{1 - 2\zeta^2/M^2}{2\sqrt{M^2 + k^2}}. \quad (4.116)$$

Thus we see that in the particular case where  $\zeta = 0$  at all times (a straight trajectory) one has  $|v_-^N(k)|^2 = 0$  and  $|v_+^T(k)|^2 = 0$  and one recovers the standard results describing the quantisation of a massless scalar field  $v_-^T$  and a massive scalar field  $v_+^N$  of mass  $M$ . Observe that in this case it was not necessary to choose (eqs. 4.65-4.67) as initial conditions to ensure the quantisation of the system.

#### 4.4.4 Low energy effective theory

Although in general it is not possible to solve (eq. 4.96) and (eq. 4.97) analytically, we may integrate the heavy mode to deduce a reliable low energy effective theory describing the light degree of freedom parallel to the trajectory as long as  $\zeta \ll M$  and  $k \ll M$ . From the discussion of the previous section, it is not difficult to anticipate that the two modes  $\alpha = 1, 2$  will be closely related to the light and heavy direction. Let us therefore adopt the notation  $\alpha = L, H$  and focus on the light mode  $v_L^I$ , which here we express as

$$v_L^I \rightarrow \begin{pmatrix} v_L^T \\ v_L^N \end{pmatrix} \equiv \begin{pmatrix} \psi \\ \chi \end{pmatrix}, \quad (4.117)$$

where  $\chi$  is a contribution satisfying  $|\ddot{\chi}| \ll M^2|\chi|$ , that is, its time variation is much slower than the time scale  $M^{-1}$  characterising the heavy mode. Then, inserting (eq. 4.117) back into the second equation of motion (eq. 4.97) and keeping the leading term in  $\chi$ , we obtain the result

$$\chi = \frac{2\zeta}{M^2}\dot{\psi} + 2\frac{V_\phi}{M^2\kappa}\psi. \quad (4.118)$$

Of course, we are due to verify that  $|\ddot{\chi}| \ll M^2|\chi|$  is a good ansatz for the solution. Inserting (eq. 4.118) back into the first equation of motion (eq. 4.96) we obtain

$$\ddot{\psi} + 4\frac{d}{dt}\left(\frac{\zeta^2}{M^2}\dot{\psi}\right) + (k^2 + m_L^2)\psi = 0, \quad (4.119)$$

$$m_L^2 = \nabla_\phi \left[ \left(1 + \frac{4\zeta^2}{M^2}\right) V_\phi \right]. \quad (4.120)$$

Simple inspection of this equation shows that indeed  $|\ddot{\chi}| \ll M^2|\chi|$  is satisfied. Additionally, from (eq. 4.118) notice that the vector (eq. 4.117) is pointing almost entirely towards the direction (1,0), which corresponds to the direction parallel to the motion of the background field. To deal with the previous equation we define  $e^\beta =$

$1 + 4\zeta^2/M^2$ .<sup>15</sup> Then, we may write

$$e^\beta (\ddot{\psi} + \dot{\beta}\dot{\psi}) + (k^2 + m_L^2)\psi = 0, \quad (4.121)$$

$$m_L^2 = \nabla_\phi (e^\beta V_\phi). \quad (4.122)$$

We can alternatively rewrite  $m_L^2 = \nabla_\phi (e^\beta V_\phi) = e^\beta \nabla_\phi V_\phi + e^\beta V_\phi \dot{\beta}/\dot{\phi}_0$ . It is possible to see that the previous equation of motion can be obtained from the action

$$S = \frac{1}{2} \int dt d^3x \left[ e^\beta \dot{\psi}^2 - (\nabla\psi)^2 - m_L^2 \psi^2 \right]. \quad (4.123)$$

By performing a field redefinition  $\varphi \equiv e^{\beta/2}\psi$ , we see that the previous action may be re-expressed as

$$S = \frac{1}{2} \int dt d^3x \left[ \dot{\varphi}^2 - e^{-\beta} (\nabla\varphi)^2 - M_L^2 \varphi^2 \right], \quad (4.124)$$

$$M_L^2 = \nabla_\phi V_\phi + \frac{V_\phi \dot{\beta}}{\dot{\phi}_0} + \frac{\ddot{\beta}}{2} + \frac{\dot{\beta}^2}{4}. \quad (4.125)$$

For the particular case in which  $\nabla_\phi V_\phi = 0$  and the bending of the trajectory is such that  $\dot{\beta} = 0$ , then the frequency  $\omega$  of the light mode reduces to  $\omega = ke^{-\beta/2} \simeq k(1 - 2\zeta^2/M^2)$ , which coincides with the previous result (eq. 4.111).

## 4.5 Discussion

In this chapter, we considered the structure of scalar field theories with a pronounced hierarchy of mass scales. First, we set up a framework for describing a light field moving along a multi-field trajectory in field space. From this, we determined the background equations of motion, around which we can study perturbations. Finally, we deduced the effective theory describing light perturbations for the case in which the background field is following a curved trajectory in field space.

The main manifestation of the non-trivial mixing of the heavy and the light directions is in the appearance of the coefficient  $e^{-\beta}$  in front of the term  $(\nabla\varphi)^2$  containing spatial derivatives appearing in the action (eq. 4.124). Since  $\beta \geq 0$ , the net effect of the bending of the background trajectory is to reduce the energy per scalar field quantum. This is due to the fact that during bending the light modes momentarily start exciting heavy modes, therefore transferring energy to them. Yet, since  $\beta = 4\zeta^2/M^2$ ,

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<sup>15</sup>Since  $\zeta^2/M^2 \ll 1$ , then  $\beta \simeq 4\zeta^2/M^2$ .

there are two effects competing against each other in this process. On one hand one has  $\zeta = \dot{\phi}/\kappa$ , which may be interpreted as the angular speed of the background field along the curved trajectory. On the other hand, there is the mass of the heavy mode  $M$ , which must be excited by the light modes during the bending. In what follows we discuss two applications of our results.

### 4.5.1 Inflation

Understanding in detail how light and heavy modes remain coupled under more general circumstances could be particularly significant for cosmic inflation (Guth, 1981, Albrecht and Steinhardt, 1982, Linde, 1982). Indeed, although current observations are consistent with the simplest model of single-field inflation, it is rather hard to conceive a realistic model where the inflaton field alone is completely decoupled from UV degrees of freedom. One way of addressing this issue is by studying multi-field scenarios where many scalar fields have the chance to participate in the inflationary dynamics (Starobinsky, 1985), despite of different mass scales among the inflaton candidates. Hence, there could exist certain phenomena related to inflation in which the effects studied in this chapter can be relevant. This has also recently been considered in Tolley and Wyman (2010) and Chen and Wang (2010b).

First, note that the equation of motion deduced out of the action (eq. 4.124) is given by

$$\ddot{\varphi} + e^{-\beta} k^2 \varphi + M_L^2 \varphi = 0 . \quad (4.126)$$

For definiteness, let us focus on phenomena characterised by  $|\dot{\beta}| \ll k$  and consider the case in which the potential is flat enough so that  $M_L^2 \ll k^2$  is satisfied. Then, the time variation of  $\beta$  along the trajectory is small enough to allow us to write the mode solution as

$$\varphi(k, t) = \frac{e^{\beta/4}}{\sqrt{k}} \exp \left[ i e^{-\beta/2} k t \right] , \quad (4.127)$$

where the factor  $e^{\beta/4}/\sqrt{k}$  is necessary in order to satisfy the commutation relation  $[\varphi, \dot{\varphi}] = i$ . This factor coincides with the one found in (eq. 4.113) for the amplitude of light modes in the case where  $\beta$  is a constant. To continue, from (eq. 4.127) we can see that in the vacuum, the two point correlation function of the perturbation  $\varphi(\mathbf{x}, t)$ , has the form  $\langle \varphi(\mathbf{x}, t) \varphi(\mathbf{y}, t) \rangle \propto e^{\beta(t)/2}$ . One direct consequence of this result is for inflation, where the amplitudes of scalar fluctuations freeze after crossing the horizon, i.e. when the physical wavelength  $k^{-1}$  satisfies the condition  $e^{-\beta/2} k = H$ . More precisely, if we generalise (eq. 4.126) to include gravity, we would conclude

that the speed of sound of adiabatic perturbations is given by

$$c_s^2 = e^{-\beta} . \quad (4.128)$$

Such an effect is known to produce sizable levels of nongaussianities (Alishahiha et al., 2004). Additionally it modifies the power spectrum as

$$P(k) \simeq e^{\beta(k)/2} P_*(k) , \quad (4.129)$$

where  $P_*(k)$  is the conventional power spectrum  $P_*(k) \propto k^{n_s-1}$  deduced in single-field slow-roll inflation, and  $\beta(k)$  is the value of  $\beta(t)$  at the time  $t$  when the mode  $k$  crosses the horizon.<sup>16</sup> The more interesting case is a varying  $\beta$  and since  $\beta$  can be as large as  $\epsilon$ , such an effect may be sizable and observable in the near future. In many scalar field theories, such as supergravity, the masses of heavy degrees of freedom during inflation are typically of order  $M \sim H$ , leading to the relation

$$\beta \sim 4\epsilon \frac{M_{\text{Pl}}^2}{\kappa^2} . \quad (4.130)$$

If the bending is such that the radius of curvature becomes of order  $\kappa \sim M_{\text{Pl}}$  (a rather conservative value) one then obtains effects as large as  $\beta \sim \epsilon$ . In the case where a turn of the trajectory happens during a few  $e$ -folds, one then should be able to observe features in the power spectrum of  $O(\epsilon)$ , particularly by modifying the running of the spectral index as  $dn_s/d \ln k$ , which otherwise would be of  $O(\epsilon^2)$ . A detailed computation of this effect is done in the next chapter.

## 4.5.2 Decoupling of light and heavy modes in supergravity

Let us next point out that our results can be also used to assess when a low energy effective theory, deduced from a multi-scalar field theory containing both heavy directions and light directions, is accurate enough. As discussed in full detail in appendix B, whenever the background fields are evolving (as in inflation) the only way of having a vanishing  $\beta$ -parameter is for a trajectory to correspond to a curve autoparallel to a geodesic in the full scalar field manifold  $\mathcal{M}$ . It is clear that the only way of achieving this is by having some property relating the shape of the potential  $V(\phi)$  with the geometry of  $\mathcal{M}$ . In the particular case of supergravity such a property is known to exist, and therefore one should expect supergravity theories rendering low

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<sup>16</sup>In the constant curvature case  $\kappa = \text{const.}$ ,  $\beta(k)$  is constant as well and we find an overall modulation of the power spectrum compatible with Chen and Wang (2010b).

energy effective theories for which  $\beta$  vanishes exactly. To be more precise, in  $\mathcal{N} = 1$  supergravity the scalar field potential is given by

$$V = e^G \left( G^{i\bar{j}} G_i G_{\bar{j}} - 3 \right), \quad (4.131)$$

with definitions matching those of section 1.3.2. Consider a supergravity theory in which a set of massive chiral fields  $\phi^H$  satisfy the condition

$$G_H = 0, \quad (4.132)$$

along a given hypersurface  $\mathcal{S}$  in  $\mathcal{M}$  parametrised *only* by the light fields  $L$ .<sup>17</sup> It is then possible to verify that the scalar fluctuations  $\phi^L$  parallel to  $\mathcal{S}$  are decoupled from the fields  $\phi^H$ , rendering  $\beta = 0$ . To appreciate this, observe first that at any point on the surface  $\mathcal{S}$  the Kähler metric  $G_{I\bar{J}}$  is diagonal between the two sectors. Indeed, since  $G_H = 0$  holds at any point in the surface, then it must be independent of arbitrary displacements  $\delta\phi^L$  along  $\mathcal{S}$ . This implies that

$$\partial_{\bar{L}} G_H = G_{H\bar{L}} = 0. \quad (4.133)$$

This condition automatically ensures that in the absence of a scalar field potential, the trajectory along  $\mathcal{S}$  will be on an autoparallel curve. It remains then to verify that the potential does not imply quadratic couplings between both sectors, therefore leaving these autoparallel geodesic trajectories unmodified. Given the first derivative of the scalar potential (eq. 1.37) and using the requirement for supersymmetry (eq. 1.51), one immediately obtains  $V_H = 0$  in  $\mathcal{S}$ . Using (eq. 1.38 and 1.39), it is not difficult to notice that also  $\nabla_L V_H = \nabla_{\bar{L}} V_H = \nabla_L V_{\bar{H}} = \nabla_{\bar{L}} V_{\bar{H}} = 0$  on  $\mathcal{S}$ , which also hinge on  $G_{H\bar{L}} = 0$  and  $G_H = 0$ . Put together, these results imply that the heavy sector will not affect the light sector as long as the background trajectory tracks the geodesically generated surface  $\mathcal{S}$ .

Conversely, deviations from the condition  $G_H = 0$ , or a surface  $\mathcal{S}$  with  $H \neq \text{const.}$ , will produce interactions leading to the appearance of the coupling  $\beta$  studied in the present work. A particularly interesting example is Gallego and Serone (2009), where  $O(\epsilon)$  couplings between heavy and light fields in the superpotential result in suppressed,  $O(\epsilon^2)$  terms in the effective action for the light fields. This result was obtained by expanding about a particular  $H = \text{const.}$  configuration which, for constant light background fields, only deviates at  $O(\epsilon)$  from the true solution to the equations of motion. But along an arbitrary background  $L(t)$  the deviation will exceed  $O(\epsilon)$  for displacements  $\Delta L/\kappa > \epsilon$  (due to the  $\Gamma_{LL}^H \dot{L}^2$  term in the  $H$  equation of motion) and the corrections to the effective action discussed in this chapter become dominant.

<sup>17</sup>This means that a surface  $\mathcal{S}$  is defined by  $f(H, \bar{H}) = 0$  rather than by a function  $f(H, \bar{H}, L, \bar{L}) = 0$  (de Alwis, 2005a and chapter 2).

### 4.5.3 Consistent decoupling and autoparallel trajectories

The condition (eq. 4.132) leads to  $(\partial_L)^n (\partial_L)^m G_H = 0$  for all  $n$  and  $m$  on the hypersurface  $\mathcal{S}$ . This means, in particular, that the metric is block diagonal,  $G_{L\bar{H}} = 0$ , on  $\mathcal{S}$ . Also,  $\Gamma_{LL}^H = G_{LL\bar{H}} G^{H\bar{H}} = 0$  and the heavy direction is at a critical point as  $V_H \propto G_H = 0$ . Thereby,  $V^H = G^{H\bar{H}} V_{\bar{H}} + G^{H\bar{L}} V_{\bar{L}} = 0$  on the hypersurface  $\mathcal{S}$ .

Inserting this in the equations of motion (eq. 4.6),

$$\ddot{\phi}^L + \Gamma_{HH}^L (\dot{\phi}^H)^2 + 2\Gamma_{HL}^L \dot{\phi}^H \dot{\phi}^L + \Gamma_{LL}^L (\dot{\phi}^L)^2 + 3H\dot{\phi}^L + V^L = 0, \quad (4.134)$$

$$\ddot{\phi}^H + \Gamma_{HH}^H (\dot{\phi}^H)^2 + 2\Gamma_{HL}^H \dot{\phi}^H \dot{\phi}^L + \Gamma_{LL}^H (\dot{\phi}^L)^2 + 3H\dot{\phi}^H + V^H = 0, \quad (4.135)$$

we see that the second one is satisfied identically for  $H = \text{const.}$ , while the first one becomes a function of the light fields only,

$$\ddot{\phi}^L + \Gamma_{LL}^L (\dot{\phi}^L)^2 + 3H\dot{\phi}^L + V^L = 0. \quad (4.136)$$

Note that this derivation only holds *on*  $\mathcal{S}$ . Any quantity that depends on a region around this trajectory, such as derivatives, would see the connection in the heavy direction.

Whether the trajectory is autoparallel in terms of the real and imaginary parts of the light fields  $L$  depends on the consistently truncated Kähler function. The effective action for the light fields has to be re-expressed to a real field sigma model action which can analysed using the machinery presented in this chapter.

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## CHAPTER 5

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# Two-field models of inflation

### 5.1 Introduction

In the previous chapter we developed a framework, extending Groot Nibbelink and van Tent (2000, 2002), in which we can calculate the effects of turning field space trajectories on its perturbations. In this chapter, we will use that framework to calculate features in the inflationary power spectrum generated by a turn in the inflaton trajectory. Furthermore, we will derive an effective field theory that is valid for the large hierarchy limit  $m_H \gg H$ , and show that also in this limit there can be significant features. As we will shortly demonstrate, the parameter determining how relevant a local turn in the background inflaton trajectory is for the effective dynamics of the adiabatic mode is given by the departure from unity of the quantity  $e^\beta = 1 + 4\dot{\phi}_0^2/(\kappa^2 M^2)$ , where  $\dot{\phi}_0$  is the speed of the inflaton background field,  $\kappa$  is the radius of curvature of the curve in field space and  $M$  is the mass of the direction normal to the trajectory. Keeping in mind that during slow-roll inflation, the inflaton velocity is given by  $\dot{\phi}_0 = \sqrt{2\epsilon}M_{\text{Pl}}H$ , with  $\epsilon$  being the usual slow-roll parameter, it follows that  $e^\beta = 1 + 8\epsilon M_{\text{Pl}}^2 H^2/(\kappa^2 M^2)$ . Thus, even with  $M^2 \gg H^2$ , if the radius of curvature describing the turn is small enough, significant imprints of heavy physics on the dynamics of the adiabatic mode can arise. More generally, whenever  $e^\beta \neq 1$ , some amount of particle creation takes place that backreacts on the dynamics of the adiabatic mode. Let us not forget that in addition to the scale invariance of the power spectrum, single-field slow-roll inflation predicts that the observed CMB temperature anisotropies seeded by the curvature perturbation satisfy Gaussian statistics to a high degree of accuracy (Maldacena, 2003).

Interestingly, in the class of models examined in this work, if the normal direction to the inflaton trajectory is sufficiently massive ( $M^2 \gg H^2$ ), it is possible to compute an effective action for the adiabatic mode capturing the relevant operators of the full multi-field dynamics. This effective theory has the characteristic that the adiabatic mode propagates with a speed of sound given by

$$c_s^2 = e^{-\beta} \quad (5.1)$$

and therefore becomes a functional of (the curvature of) the trajectory traversed by the inflaton (as discussed in the previous chapter). Interestingly, this result has as a special case the particular context of Tolley and Wyman (2010) and indicates the presence of non-Gaussian signatures correlated with features in the power spectrum.<sup>1</sup>

This chapter will start with a discussion of two field models for inflation, generalising the discussion of section 4.4. In this section, we will also derive the appropriate expressions for the power spectrum. In the next section, section 5.3, we will derive a single-field effective theory with a reduced speed of sound, where the speed of sound is determined by turns in the direction of the field that is integrated out. Then, we will derive the two-field equations of motion in the limit of slow-roll inflation. We conclude this section by showing that the effective field theory can also be found in the traditional way (Rubin, 2001). Next, in section 5.4 we solve the equations of motion for specific trajectories with turns, and show that this leads to oscillations in the power spectrum. We also perform the calculation in the effective single-field theory with a reduced speed of sound, and see that for the large hierarchy limit both methods agree. This leads to the conclusion that the effect of a turn in field space is signalled by a simultaneous appearance of oscillations in the power spectrum and nongaussianities, as is explained in section 5.5

## 5.2 Inflationary models with two scalar fields

We now study the evolution of perturbations in systems containing only two relevant scalar fields. In this case, it is always possible to take the set of vielbeins  $\{e_I^a\}$  to consist entirely in  $e_T^a = T^a$  and  $e_N^a = N^a$  defined in Section 4.2.1. Then, the projection tensor  $P_{ab}$  introduced in (eq. 4.23) vanishes identically and one is left with the

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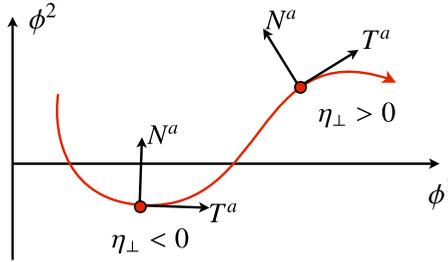
<sup>1</sup>In the paper by Cremonini et al. (2010b) the parametrisation of the non-decoupling parameter of the isocurvature directions  $\xi$  relates as a specific realisation of our analysis. This is easiest seen through comparing expressions (23) in Cremonini et al. (2010b) with (eq. 5.37) or (eq. 5.60) here.

relations

$$\frac{DT^a}{dt} = -H\eta_{\perp}N^a, \quad (5.2)$$

$$\frac{DN^a}{dt} = H\eta_{\perp}T^a. \quad (5.3)$$

At this point we notice that the normal vector  $N^a$  has always the same orientation with respect to the curved trajectory, which is due to the presence of the signature function  $s_N$  in (eq. 4.9). For definiteness, let us agree that the normal direction  $N^a$  has a right-handed orientation with respect to  $T^a$  as shown in Figure 5.1. With this



**Figure 5.1:** The figure shows a fixed right-handed orientation of  $N^a$  with respect to  $T^a$ . If the turn is towards the left then  $\eta_{\perp}$  is negative, whereas if the turn is towards the right then  $\eta_{\perp}$  is positive.

convention  $\eta_{\perp}$  changes signs smoothly in such a way that if the turn is towards the left then  $\eta_{\perp}$  is negative, whereas if the turn is towards the right then  $\eta_{\perp}$  is positive. A concrete choice for  $T^a$  and  $N^a$  with these properties is:

$$T^a = \frac{1}{\dot{\phi}_0} (\dot{\phi}^1, \dot{\phi}^2), \quad (5.4)$$

$$N^a = \frac{1}{\dot{\phi}_0 \sqrt{\gamma}} (-\gamma_{22}\dot{\phi}^2 - \gamma_{12}\dot{\phi}^1, \gamma_{11}\dot{\phi}^1 + \gamma_{21}\dot{\phi}^2), \quad (5.5)$$

where  $\gamma = \gamma_{11}\gamma_{22} - \gamma_{12}\gamma_{21}$  is the determinant of  $\gamma_{ab}$ . To continue, parallel and normal perturbations with respect to the inflationary trajectory are then given by

$$v^T = a Q^T = a T_a Q^a, \quad (5.6)$$

$$v^N = a Q^N = a N_a Q^a. \quad (5.7)$$

By choosing this frame, one finds that  $Z_{TN} = -Z_{NT} = aH\eta_{\perp}$ . The coupled equations of motion describing the evolution of both modes  $v_{\alpha}^T(k, \tau)$  and  $v_{\alpha}^N(k, \tau)$  become

$$\frac{d^2 v_{\alpha}^T}{d\tau^2} + 2\zeta \frac{dv_{\alpha}^N}{d\tau} - \zeta^2 v_{\alpha}^T + \frac{d\zeta}{d\tau} v_{\alpha}^N + \Omega_{TN} v_{\alpha}^N + (\Omega_{TT} + k^2) v_{\alpha}^T = 0, \quad (5.8)$$

$$\frac{d^2 v_{\alpha}^N}{d\tau^2} - 2\zeta \frac{dv_{\alpha}^T}{d\tau} - \zeta^2 v_{\alpha}^N - \frac{d\zeta}{d\tau} v_{\alpha}^T + \Omega_{NT} v_{\alpha}^T + (\Omega_{NN} + k^2) v_{\alpha}^N = 0, \quad (5.9)$$

where we have defined

$$\zeta \equiv Z_{TN} = aH\eta_{\perp}. \quad (5.10)$$

In the previous equations, the symmetric matrix  $\Omega_{IJ}$  is defined in (eq. 4.36) and (eq. 4.48) and consists of the following elements:

$$\Omega_{TT} = -a^2 H^2 (2 + 2\epsilon - 3\eta_{\parallel} + \eta_{\parallel} \xi_{\parallel} - 4\epsilon \eta_{\parallel} + 2\epsilon^2 - \eta_{\perp}^2), \quad (5.11)$$

$$\Omega_{NN} = -a^2 H^2 (2 - \epsilon) + a^2 M^2, \quad (5.12)$$

$$\Omega_{TN} = a^2 H^2 \eta_{\perp} (3 + \epsilon - 2\eta_{\parallel} - \xi_{\perp}), \quad (5.13)$$

where  $M^2 \equiv V_{NN} + H^2 M_{\text{pl}}^2 \epsilon \mathbb{R}$  is the effective squared mass of the  $v^N$ -mode and  $\mathbb{R} = 2\mathbb{R}_{TNTN} = T^a N^b T^c N^d \mathbb{R}_{abcd}$  is the Ricci scalar parametrising the geometry of  $\mathcal{M}$ . Furthermore,  $\xi_{\parallel}$  was defined in (eq. 4.31) and additionally we have defined<sup>2</sup>

$$\xi_{\perp} \equiv -\frac{\dot{\eta}_{\perp}}{H\eta_{\perp}}. \quad (5.14)$$

To arrive at the form of the mass matrix  $\Omega_{IJ}$  shown in (eq. 5.11), (eq. 5.12) and (eq. 5.13), we may start from the explicit form deduced out of (eq. 4.36) and (eq. 4.48) for the case of two-field models,

$$\Omega_{TT} = -a^2 H^2 (2 - \epsilon) + a^2 V_{\phi\phi} - 2a^2 H^2 \epsilon (3 - 2\eta_{\parallel} + \epsilon), \quad (5.15)$$

$$\Omega_{NN} = -a^2 H^2 (2 - \epsilon) + a^2 V_{NN} + a^2 H^2 M_{\text{pl}}^2 \epsilon \mathbb{R}, \quad (5.16)$$

$$\Omega_{TN} = a^2 V_{\phi N} + 2a^2 H^2 \eta_{\perp} \epsilon, \quad (5.17)$$

where we have defined

$$V_{\phi\phi} \equiv T^a T^b \nabla_a V_b, \quad (5.18)$$

$$V_{NN} \equiv N^a N^b \nabla_a V_b, \quad (5.19)$$

$$V_{\phi N} \equiv T^a N^b \nabla_a V_b. \quad (5.20)$$

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<sup>2</sup>Note that this definition for  $\xi_{\perp}$  is different from the definition used in Groot Nibbelink and van Tent (2000, 2002).

Additionally  $\mathbb{R} = 2\mathbb{R}_{TNTN} = T^a N^b T^c N^d \mathbb{R}_{abcd}$  is the Ricci scalar parametrising the geometry of  $\mathcal{M}$ . Notice that  $V_{\phi\phi}$  can be rewritten in the following way:

$$\begin{aligned} V_{\phi\phi} &= T^a \nabla_a (T^b V_b) - T^a (\nabla_a T^b) V_b \\ &= \nabla_\phi V_\phi - \frac{1}{\dot{\phi}_0} \frac{DT^b}{dt} V_b \\ &= \nabla_\phi V_\phi + H^2 \eta_\perp^2, \end{aligned} \tag{5.21}$$

where, to go from the second to the third line we made use of (eq. 5.2) and relation  $V_N = \dot{\phi}_0 H \eta_\perp$  coming from the definition of  $\eta_\perp$  in (eq. 4.18). Similarly, the quantity  $V_{\phi N}$  may be manipulated as follows:

$$\begin{aligned} V_{\phi N} &= T^a \nabla_a (N^b V_b) - T^a (\nabla_a N^b) V_b \\ &= \nabla_\phi V_N - \frac{1}{\dot{\phi}_0} \frac{DN^b}{dt} V_b \\ &= \nabla_\phi V_N - \frac{H \eta_\perp}{\dot{\phi}_0} V_\phi, \end{aligned} \tag{5.22}$$

where again, to go from the second to the third line, we made use of (eq. 5.2). As a final step, we may use  $V_N = \dot{\phi}_0 H \eta_\perp$  to deduce

$$\nabla_\phi V_N = \frac{1}{\dot{\phi}_0} \frac{d}{dt} (\dot{\phi}_0 H \eta_\perp) = -H^2 \eta_\perp (\eta_\parallel + \epsilon + \xi_\perp). \tag{5.23}$$

Collecting all of these terms back into (eq. 5.15), (eq. 5.16) and (eq. 5.17) we finally arrive at (eq. 5.11), (eq. 5.12) and (eq. 5.13). Observe that we are not able to rewrite  $V_{NN} = N^a N^b \nabla_a V_b$  in a similar way, since it involves second variations away from the inflationary trajectory. This simply means that the quantity  $V_{NN}$  must be regarded as an additional parameter of the model related to the mass of the transverse mode with respect to the inflaton trajectory.

### 5.2.1 Power spectrum

Expressions (eq. 5.8) and (eq. 5.9) consist of the equations of motion necessary to deduce the generation of the curvature perturbation in the case of two-field inflation. Once the solutions of the fields  $v^T = a Q^T$  and  $v^N = a Q^N$  are known, it is possible to

define the curvature and isocurvature perturbations as

$$\mathcal{R} \equiv \frac{H}{\dot{\phi}_0} Q^T, \quad (5.24)$$

$$\mathcal{S} \equiv \frac{H}{\dot{\phi}_0} Q^N, \quad (5.25)$$

respectively. Using equation (eq. 4.74) with  $I = J = T$ , the resulting power spectrum for adiabatic modes are found to be

$$\mathcal{P}_{\mathcal{R}}(k, \tau) = \frac{H^2}{\dot{\phi}_0^2} \mathcal{P}_Q^{TT}(k, \tau) = \frac{k^3}{4\pi^2 a^2 M_{\text{Pl}}^2 \epsilon} \sum_{\alpha=1,2} v_{\alpha}^T(k, \tau) v_{\alpha}^{T*}(k, \tau), \quad (5.26)$$

where  $a$  and  $\epsilon = \dot{\phi}^2 / (2M_{\text{Pl}}^2 H^2)$  are functions of  $\tau$ . We can also compute the power spectrum for isocurvature modes and cross correlation as (Gordon et al., 2001, Amendola et al., 2002, Wands et al., 2002)

$$\mathcal{P}_{\mathcal{S}}(k, \tau) = \frac{H^2}{\dot{\phi}_0^2} \mathcal{P}_Q^{NN}(k, \tau), \quad (5.27)$$

$$\mathcal{P}_{\mathcal{RS}}(k, \tau) = \frac{H^2}{\dot{\phi}_0^2} \mathcal{P}_Q^{TN}(k, \tau), \quad (5.28)$$

respectively. They can give rise to observable signatures in the CMB power spectrum (Amendola et al., 2002), but it depends on post-inflationary processes which we do not consider here. In this work we are primarily concerned with the computation of the power spectrum of the curvature perturbation  $\mathcal{R}$  at the end of inflation. This corresponds to the quantity

$$\mathcal{P}_{\mathcal{R}}(k) \equiv \mathcal{P}_{\mathcal{R}}(k, \tau_{\text{end}}), \quad (5.29)$$

where  $\tau_{\text{end}}$  is the time at which inflation effectively ends.<sup>3</sup> The computations of  $\mathcal{P}_{\mathcal{S}}$  and  $\mathcal{P}_{\mathcal{RS}}$  can be done in an identical way.

## 5.2.2 Effective Theory

If a hierarchy of scales is present in the matrix  $\Omega_{IJ}$ , then we can compute a fairly reliable effective theory out of the system (eq. 5.8) and (eq. 5.9). Indeed, by assuming

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<sup>3</sup>Since in multi-field inflation the adiabatic mode  $\mathcal{R}$  (as well as other background quantities) may continue evolving on super horizon scales, here we do not follow the standard practise of evaluating the power spectrum at horizon crossing time  $k = aH$  (Gong and Stewart, 2002). See Kinney (2005) for a discussion of this point.

that  $\Omega_{NN}$  remains positive at all times and that

$$\Omega_{NN} \gg |\Omega_{TT}|, \quad (5.30)$$

$$\Omega_{NN} \gg |\Omega_{TN}|, \quad (5.31)$$

we may integrate the heavy mode  $v^N$  out of the system of equations. By examining the specific shape of the entries  $\Omega_{TT}$ ,  $\Omega_{NN}$  and  $\Omega_{TN}$ , we see that a generic requisite for this hierarchy to exist is

$$M^2 \gg H^2, \quad (5.32)$$

where  $M^2$  is the effective mass of the heavy mode  $v^N$  given by

$$M^2 \equiv V_{NN} + H^2 M_{\text{pl}}^2 \in \mathbb{R}. \quad (5.33)$$

To compute the effective theory we proceed in the same way as in chapter 4. We focus on the mode  $\alpha$  associated to slower oscillations due to the hierarchy. Omitting the  $\alpha$  label, this mode is necessarily such that

$$\left| \frac{d^2 v^N}{d\tau^2} \right| \ll a^2 M^2 v^N. \quad (5.34)$$

This allows us to disregard the second derivative of  $v^N$  in (eq. 5.9), and write  $v^N$  in terms of  $v^T$  as

$$v^N = \frac{1}{\Omega_{NN} - \zeta^2 + k^2} \left( 2\zeta \frac{dv^T}{d\tau} + \frac{d\zeta}{d\tau} v^T - \Omega_{NT} v^T \right). \quad (5.35)$$

This expression for  $v^N$  can be inserted back into the remaining equation of motion (eq. 5.8) to obtain an effective equation of motion for the light adiabatic mode  $v^T$ . Then, by defining a new field  $\varphi$  as

$$\varphi \equiv e^{\beta/2} v^T, \quad (5.36)$$

$$e^{\beta(\tau, k^2)} \equiv 1 + 4\eta_{\perp}^2 \left( \frac{M^2}{H^2} - 2 + \epsilon - \eta_{\perp}^2 + \frac{k^2}{a^2 H^2} \right)^{-1}. \quad (5.37)$$

we finally arrive at the following effective equation of motion

$$\varphi'' + e^{\beta(\tau, k^2)} k^2 \varphi + \Omega(\tau, k^2) \varphi = 0, \quad (5.38)$$

where the time dependent function  $\Omega(\tau, k^2)$  is found to be

$$\Omega(\tau, k^2) = \Omega_0(\tau) - \frac{\beta''}{2} - \left( \frac{\beta'}{2} \right)^2 - aH\beta'(1 + \epsilon - \eta_{\parallel}), \quad (5.39)$$

$$\Omega_0(\tau) = -a^2 H^2 (2 + 2\epsilon - 3\eta_{\parallel} - 4\epsilon\eta_{\parallel} - \xi_{\parallel}\eta_{\parallel} + 2\epsilon^2). \quad (5.40)$$

Notice that  $\Omega_0$  is precisely the mass term appearing in the conventional equation of motion for adiabatic fluctuations in single-field slow-roll inflation. Furthermore, we note that in the case where the mass  $M$  approaches the cutoff of our theory, our results can be derived from an effective action for the adiabatic mode given by the action

$$S = \frac{1}{2} \int d\tau d^3x \left[ \left( \frac{d\varphi}{d\tau} \right)^2 - \nabla\varphi e^{-\beta(\tau, -\nabla^2)} \nabla\varphi - \varphi \Omega(\tau, -\nabla^2) \varphi \right], \quad (5.41)$$

where  $\beta(\tau, -\nabla^2)$  and  $\Omega(\tau, -\nabla^2)$  are the functions defined in (eqs. 5.37 and 5.39) but with  $k^2$  replaced by  $-\nabla^2$ . This result corresponds to the generalisation of our previous work, discussed in chapter 4, to the case of a slowly rolling background in the presence of gravity. A slightly more formal deduction of this effective theory may be found in section 5.3.4, where we see that it can be viewed as a leading order effect at the loop level, and as such contains the higher dimensional corrections implied by the general arguments made in Weinberg (2008) and Cheung et al. (2008). In section 5.4 we shall compare the power spectrum obtained using this effective theory with the one obtained from the full set of equations for the perturbations. We anticipate that this effective theory is very reliable regardless of how large the values of  $\beta$  are.

## 5.3 Slow-roll inflation in two-field models

So far we have not assumed the slow evolution of background quantities. We now proceed to discuss the case of inflation realised in the slow-roll regime, where the scale of inflation  $H$  varies slowly. Our main interest is to study the effects appearing from curved inflationary trajectories, where  $\eta_\perp$  is non-vanishing. We will assume that the radius of curvature  $\kappa$  may take values smaller than  $M_{\text{Pl}}$ , corresponding to turns of the trajectory taking place at field scales smaller than the Planck scale. This situation is certainly allowed and depending on the value of  $\epsilon$  it may render large values of  $\eta_\perp$  (recall (eq. 4.22) relating  $\eta_\perp$  and  $\kappa$ ). By the same token, we will consider models where the normal mode  $\nu^N$  has a large effective mass  $M^2 \gg H^2$ .

### 5.3.1 Slow-roll parameters

In general, given the background equations of motion (eq. 4.6-4.8), we say that a given background quantity  $A$  is slowly rolling if its variation satisfies

$$|\delta_A| \equiv \left| -\frac{1}{HA} \frac{dA}{dt} \right| \ll 1. \quad (5.42)$$

Observe that we can write  $\epsilon = \delta_H$  and  $\eta_{\parallel} = \delta_{\dot{\phi}_0}$ , and therefore both  $H$  and  $\dot{\phi}_0$  evolve slowly if  $\epsilon \ll 1$  and  $|\eta_{\parallel}| \ll 1$  respectively. Since  $\epsilon = \dot{\phi}^2/(2M_{\text{pl}}^2 H^2)$ , the condition  $|\eta_{\parallel}| \ll 1$  also guaranties that  $\epsilon$  will remain varying slowly during inflation. It is useful to introduce a single small dimensionless number  $\delta \ll 1$  parametrising the slow-roll expansion<sup>4</sup> and demand that any quantity  $A$  to which slow-roll is imposed, generically satisfies

$$\frac{1}{HA} \frac{dA}{dt} = \mathcal{O}(\delta), \quad (5.43)$$

which means  $\epsilon = \mathcal{O}(\delta)$  and  $\eta_{\parallel} = \mathcal{O}(\delta)$ . In the absence of clear evidence of it, for simplicity we shall not consider here hierarchies between different slow-roll parameters. Recall that (eqs. 4.29 and 4.30) are exact equations relating the parameters  $\epsilon$ ,  $\eta_{\parallel}$  and  $\xi_{\parallel}$  to the shape of the potential  $V$  along the inflationary trajectory. Now, provided that all of these parameters are small, we may re-express these equations to leading order in  $\delta$  as

$$\eta_{\parallel} + \epsilon = M_{\text{pl}}^2 \frac{\nabla_{\phi} V_{\phi}}{V}, \quad (5.44)$$

$$\epsilon = \frac{M_{\text{pl}}^2}{2} \left( \frac{V_{\phi}}{V} \right)^2. \quad (5.45)$$

These are the usual equations defining the slow-roll parameters in terms of the shape of the first and second derivatives of  $V$ .<sup>5</sup> As long as  $\epsilon \ll 1$  and  $|\eta_{\parallel}| \ll 1$ , the background geometry evolves slowly and the scalar field velocity is determined by the attractor equation of motion  $3H\dot{\phi}_0 + V_{\phi} = 0$ . For completeness, notice from the definition of  $\eta_{\perp}$  in (eq. 4.18) that it is possible to write  $\eta_{\perp} = V_N/(\sqrt{2}\epsilon M_{\text{pl}} H^2)$ . Then, using (eq. 5.45) we deduce

$$\eta_{\perp}^2 = 9 \left( \frac{V_N}{V_{\phi}} \right)^2, \quad (5.46)$$

which is valid to leading order in  $\delta$ . This equation nicely relates the slope of the potential  $V_{\phi}$  along the tangential direction  $T^a$  with its counterpart  $V_N$  along the normal direction  $N^a$ .

<sup>4</sup>Current observations indicate that the order of such a reference parameter is given by the departure of the spectral index from unity  $\delta \sim |n_{\mathcal{R}} - 1|$ .

<sup>5</sup>Let us recall that the parameter  $\eta$  was originally introduced in the study of single-field slow-roll inflation (Liddle and Lyth, 1992) as  $\eta = M_{\text{pl}}^2 V''/V$ . Therefore, in order to compare the present results with those following the original convention, we must write  $\eta = \eta_{\parallel} + \epsilon$ .

### 5.3.2 Perpendicular dynamics

Let us now turn our attention to parameter  $\eta_\perp$  defined in (eq. 4.18). Notice that this parameter is *not* related to the slow-roll variation of any given background quantity  $A$  in the sense of (eq. 5.42) and therefore is not constrained to be of  $\mathcal{O}(\delta)$ . Moreover, (eq. 4.22) tells us that  $\eta_\perp$  may be large compared to  $\delta$  provided that the radius of curvature  $\kappa$  is small compared to  $\sqrt{2\epsilon}M_{\text{Pl}}$ . It is important to recognise that the curved inflationary trajectory ( $\kappa^{-1} \neq 0$ ) has its origin in both the shape of the scalar potential  $V$  and the geometry of the scalar manifold where the theory lives. In particular, since  $H$  and  $\dot{\phi}_0$  are assumed to evolve slowly, we expect the flat inflationary trajectory to remain close to the locus of points minimising the heaviest direction  $N^a$  of the potential. In other words, to ensure a bending of the trajectory we consider models where the potential is such that

$$V_{NN} \gg |\nabla_\phi V_\phi|. \quad (5.47)$$

It is entirely clear that in the event that the inflationary trajectory is suffering a turn, it will not coincide exactly with curve minimising the heaviest direction, which is made explicit by the result  $V_N = \eta_\perp \dot{\phi}_0 H$  found in (eq. 4.18). It is in fact easy to show that the departure  $\Delta$  from the real minima  $V_N|_{\text{min}} = 0$  is roughly given by the condition  $V_N + M^2 \Delta \simeq 0$ , with  $M^2$  given by (eq. 5.33). Then, with the help of (eq. 4.18) one finds that the ratio between the deviation  $\Delta$  and the radius of curvature  $\kappa$  is given by

$$\frac{\Delta}{\kappa} \simeq \eta_\perp^2 \frac{H^2}{M^2}. \quad (5.48)$$

Observe that  $\Delta/\kappa$  is essentially the combination  $e^\beta - 1$  defined in (eq. 5.36) in the regime  $k^2 \ll a^2 H^2$ . Thus the parameter  $\beta$  appearing in the effective theory deduced in section 5.2.2 is giving us information regarding the dynamics perpendicular to the inflaton trajectory.

It is important to check whether the bending interferes with the flatness of the potential as felt by the adiabatic mode  $v^T$ . Observe from (eq. 5.8) and (eq. 5.11) that the effective mass  $m^2(\tau)$  of  $v^T$  is given by

$$m^2(\tau) \equiv \Omega_{TT} - \zeta^2 \approx -a^2 H^2 (2 + 2\epsilon - 3\eta_\parallel), \quad (5.49)$$

where we have neglected terms of  $\mathcal{O}(\delta^2)$ . Note that  $m^2(\tau) = \Omega_0(\tau)$ , where  $\Omega_0(\tau)$  is the effective mass encountered in the effective theory deduced in section 5.2.2. Thus, we see that  $\eta_\perp$  does not directly spoil the flatness of the potential  $V$ . Of course, one should explicitly verify in which way a bending affects the value of  $\epsilon$  and  $\eta_\parallel$  by

examining the evolution of the background. We however point out that there is no reason *a priori* that fast and sudden turns with large values of  $\eta_\perp$  are not possible while staying in the slow-roll regime.

### 5.3.3 Equations of motion in the slow-roll regime

Putting all of the previous results together back into the set of equations (eq. 5.8) and (eq. 5.9), and neglecting terms of  $\mathcal{O}(\delta^2)$ , we finally arrive at the following equations of motion for the perturbations  $v_\alpha^T$  and  $v_\alpha^N$ :

$$\frac{d^2 v_\alpha^T}{d\tau^2} + 2aH\eta_\perp \frac{dv_\alpha^N}{d\tau} + a^2 H^2 \left( \frac{k^2}{a^2 H^2} - 2 - 2\epsilon + 3\eta_\parallel \right) v_\alpha^T + 2a^2 H^2 \eta_\perp (2 - \xi_\perp) v_\alpha^N = 0, \quad (5.50)$$

$$\frac{d^2 v_\alpha^N}{d\tau^2} - 2aH\eta_\perp \frac{dv_\alpha^T}{d\tau} + a^2 H^2 \left( \frac{k^2}{a^2 H^2} + \frac{M^2}{H^2} - 2 + \epsilon - \eta_\perp^2 \right) v_\alpha^N + 2a^2 H^2 \eta_\perp v_\alpha^T = 0, \quad (5.51)$$

where  $\xi_\perp$  was defined in (eq. 5.14). In the next section we deal with these equations numerically for suitable choices of the background parameters, and compare the obtained power spectrum with that of the effective theory obtained in section 5.2.2. We shall see how features in the power spectrum appear as a consequence of curved inflationary trajectory. We will, however, first give another derivation of the effective theory presented in section 5.2.2.

### 5.3.4 Effective theory for the adiabatic mode

In this section we offer another deduction of the effective theory shown in section 5.2.2. We begin by writing the action (eq. 4.49) for the particular case of two fields:

$$\begin{aligned} S = & \int d\tau d^3x \frac{1}{2} \left[ \left( \frac{dv^T}{d\tau} \right)^2 - (\nabla v^T)^2 - (\Omega_{TT} - \zeta^2) (v^T)^2 \right] \\ & + \int d\tau d^3x \frac{1}{2} \left[ \left( \frac{dv^N}{d\tau} \right)^2 - (\nabla v^N)^2 - (\Omega_{NN} - \zeta^2) (v^N)^2 \right] \\ & - \int d\tau d^3x v^N \left( \Omega_{TN} - \frac{d\zeta}{d\tau} - 2\zeta \frac{d}{d\tau} \right) v^T. \end{aligned} \quad (5.52)$$

Given that  $\Omega_{NN} \gg |\Omega_{TT}|$  and  $\Omega_{NN} \gg |\Omega_{TN}|$  the field  $v^N$  is the heavier of the two. Taking this scale as the scale of the heavy physics that we wish to integrate out, we

can formally evaluate the functional integral for  $v^N$  to obtain the one loop effective action for  $v^T$  as

$$S = \int d\tau d^3x \frac{1}{2} \left[ \left( \frac{dv^T}{d\tau} \right)^2 - (\nabla v^T)^2 - (\Omega_{TT} - \zeta^2) (v^T)^2 \right] + \frac{1}{2} \int d\tau d^3x \int d\tau' d^3x' \mathcal{O}(\tau) v^T(\mathbf{x}, \tau) G(\mathbf{x}, \tau; \mathbf{x}', \tau') \mathcal{O}(\tau') v^T(\mathbf{x}', \tau') + S_{CT}, \quad (5.53)$$

with  $\mathcal{O}$  given by

$$\mathcal{O}(\tau) \equiv - \left( \Omega_{TN} - \frac{d\zeta}{d\tau} - 2\zeta \frac{d}{d\tau} \right), \quad (5.54)$$

and the Green's function  $G$  given by

$$G(x, \tau; x', \tau') = \frac{1}{\square + \Omega_{NN} - \zeta^2}. \quad (5.55)$$

The term  $S_{CT}$  renormalises the effective action for the background inflaton field. We have to demand that the parameters of this effective action that satisfy the slow-roll conditions rather than those of the bare action (Burgess et al., 2010), which we presume to be the case here. In general, evaluating the full effective action is a highly non-trivial task. However, in Fourier space one can formally make the expansion

$$G(\tau, \tau', k) = \frac{1}{-\partial_\tau^2 + k^2 + \Omega_{NN} - \zeta^2} = \frac{1}{\omega^2} \left( 1 - \frac{\partial_\tau^2}{\omega^2} + \dots \right), \quad (5.56)$$

where

$$\omega^2 \equiv k^2 + \Omega_{NN} - \zeta^2. \quad (5.57)$$

Where implicit in the above is that if the scale  $M$  tends to the cutoff of the theory (so that  $V_{NN} \sim M^2$ ) we can neglect the temporal derivatives in the expansion above, relative to the mass term and the spatial derivatives (which always become significant at horizon crossing). This reduces the Green's function to leading order of only the contact term.<sup>6</sup> Integrating the second term in (5.53) by parts results in

$$S = \int d\tau d^3k \frac{1}{2} \left\{ \left( \frac{dv^T}{d\tau} \right)^2 e^{\beta(k, \tau)} - \left[ k^2 + \bar{\Omega}(\tau, k) \right] (v^T)^2 \right\}, \quad (5.58)$$

---

<sup>6</sup>A related derivation for the effective field theory of the inflaton field coupled to a massive field with a cubic interaction term with the inflaton can be found in Rubin (2001).

with

$$e^{\beta(\tau, k^2)} \equiv 1 + 4\eta_{\perp}^2 \left( \frac{M^2}{H^2} - 2 + \epsilon - \eta_{\perp}^2 + \frac{k^2}{a^2 H^2} \right)^{-1}, \quad (5.59)$$

$$\bar{\Omega}(\tau, k) \equiv \Omega_0 - \frac{4a^4 H^4 \eta_{\perp}^2 (1 + \epsilon - \eta_{\parallel})^2}{\omega^2} + 4 \frac{d}{d\tau} \left[ \frac{a^3 H^3 \eta_{\perp}^2 (1 + \epsilon - \eta_{\parallel})}{\omega^2} \right], \quad (5.60)$$

whilst  $\Omega_0$  is given by (eq. 5.40). Making the field redefinition  $\varphi \equiv e^{\beta/2} v^T$  (and upon integrating by parts the resulting friction term), one then obtains the effective action

$$S = \int d\tau d^3k \frac{1}{2} \left[ \left( \frac{d\varphi}{d\tau} \right)^2 - \varphi e^{-\beta(\tau, k)} k^2 \varphi - \varphi \Omega(\tau, k) \varphi \right], \quad (5.61)$$

where  $\Omega(\tau, k^2)$  is defined as in (eq. 5.39). We thus see that the expression (eq. 5.41) follows.

## 5.4 Features in the power spectrum

We now study the evolution of perturbations and analyse how features in the primordial spectrum are generated along curved trajectories. To this extent, we solve (eqs. 5.50 and 5.51) numerically for different background solutions representing curved trajectories and obtain the mode solutions  $v_{\alpha}^I$  which, with the help of (eq. 5.29), provide us the desired power spectrum at the end of inflation. For definiteness, we consider models of inflation with an inflationary period of at least 60  $e$ -folds and set the initial conditions a few  $e$ -folds before this period starts. To avoid unnecessary complications with initial conditions, we considered models where turns in the trajectory only happen within the last 60  $e$ -folds. Before this period,  $\eta_{\perp} = 0$  and the equations of motion determining the evolution of perturbations reduce to

$$\frac{d^2 v_{\alpha}^T}{d\tau^2} + a^2 H^2 \left( \frac{k^2}{a^2 H^2} - 2 - 2\epsilon + 3\eta_{\parallel} \right) v_{\alpha}^T = 0, \quad (5.62)$$

$$\frac{d^2 v_{\alpha}^N}{d\tau^2} + a^2 H^2 \left( \frac{k^2}{a^2 H^2} + \frac{M^2}{H^2} - 2 + \epsilon \right) v_{\alpha}^N = 0. \quad (5.63)$$

Then, as long as  $\epsilon$  and  $\eta_{\parallel}$  are small, we are allowed to make use of initial conditions (eq. 4.65-4.67) with  $e_{\alpha}^I = \delta_{\alpha}^I$ , and  $v_1(k)$  and  $v_2(k)$  given by

$$v_1(k) = \frac{\sqrt{\pi}}{4\sqrt{a_i H_i}} e^{i\frac{\pi}{2}(\nu_1 + \frac{1}{2})} H_{\nu_1}^{(1)}\left(\frac{k}{a_i H_i}\right), \quad (5.64)$$

$$v_2(k) = \frac{\sqrt{\pi}}{4\sqrt{a_i H_i}} e^{i\frac{\pi}{2}(\nu_2 + \frac{1}{2})} H_{\nu_2}^{(1)}\left(\frac{k}{a_i H_i}\right), \quad (5.65)$$

where  $H_{\nu}^{(1)}(x)$  denotes the first kind Hankel function, whereas  $a_i$  and  $H_i$  are the values for the scale factor and Hubble parameter at the initial time  $\tau_i$ . Similarly, the quantities  $\pi_1(k)$  and  $\pi_2(k)$  entering the initial conditions (eq. 4.67) are given by the time derivatives of the previous expressions. On the other hand, the parameters  $\nu_1$  and  $\nu_2$  are respectively given by

$$\nu_1 = \sqrt{\frac{(3 - \epsilon)^2}{4(1 - \epsilon)^2} - 3(\eta - \epsilon)}, \quad (5.66)$$

$$\nu_2 = \sqrt{\frac{(3 - \epsilon)^2}{4(1 - \epsilon)^2} - \frac{M^2}{H_i^2}}. \quad (5.67)$$

Note that in the short wavelength limit,  $k \gg a_i H_i$ , the previous conditions matches the mode fluctuations about a Bunch-Davies vacuum (eq. 4.70), discussed in section 4.3.2. In all of the cases examined, we consider inflationary trajectories where  $\epsilon$ ,  $\eta_{\parallel}$  and  $\xi_{\parallel}$  remain small during the interval of interest, while allowing different types of time variation of  $\eta_{\perp}$ , which is the quantity that parametrises the bending.

### 5.4.1 Constant radius of curvature

Let us start by considering the simple case in which  $\eta_{\perp}$  is constant during the whole period of inflation where currently accessible modes were generated. As we have already emphasised, if  $\epsilon$  remains nearly constant a constant  $\eta_{\perp}$  corresponds to a trajectory with a constant radius of curvature  $\kappa$ . We find that the overall effect of having a constant turn is simply to normalise the amplitude of the spectrum, without modifying the usual single-field dependence of the spectral index  $n_{\mathcal{R}}$  in terms of the slow-roll parameters  $\epsilon$  and  $\eta_{\parallel}$  (see also Chen and Wang, 2010a,b),

$$n_{\mathcal{R}} - 1 = 2\eta_{\parallel} - 4\epsilon. \quad (5.68)$$

In the case  $M^2/H^2 \gg 1$ , the predicted power spectrum obtained by the effective theory is indistinguishable from the one obtained by solving the full set of equations.

Moreover, with the help of this effective theory, it is in fact possible to infer a simple relation between the power spectrum  $\mathcal{P}_{\mathcal{R}}(k)$  with  $\eta_{\perp} \neq 0$  and the analytical power spectrum  $\mathcal{P}_{\mathcal{R}}^{(0)}(k)$  computed with  $\eta_{\perp} = 0$ . To this extent, notice that although  $\beta(k, \tau)$  is a function of  $k$ , we see that when the physical wavelength of the mode becomes larger than the scale  $M^{-1}$  (i.e.  $k^2/a^2 \leq M^2$ ), the parameter  $\beta(\tau, k)$  becomes effectively  $k$  independent, and we can write

$$e^{\beta} = 1 + 4\eta_{\perp}^2 \frac{H^2}{M^2}. \quad (5.69)$$

Since  $M^2 \gg H^2$ , this happens before horizon crossing and the relevant dynamics is well described by this  $k$ -independent form of  $\beta$ . Then the relation between  $\mathcal{P}_{\mathcal{R}}(k)$  and  $\mathcal{P}_{\mathcal{R}}^{(0)}(k)$ , as predicted by the effective theory, becomes

$$\mathcal{P}_{\mathcal{R}}(k) = \left(1 + 4\eta_{\perp}^2 \frac{H^2}{M^2}\right) \mathcal{P}_{\mathcal{R}}^{(0)}(k). \quad (5.70)$$

This result modifies the usual normalisation condition of the spectrum coming from the COBE data, leading to the following relation among the various parameters:

$$\left(1 + 4\eta_{\perp}^2 \frac{H^2}{M^2}\right) \mathcal{P}_{\mathcal{R}}^{(0)}(k_{\text{COBE}}) \approx 2.46 \times 10^{-9}. \quad (5.71)$$

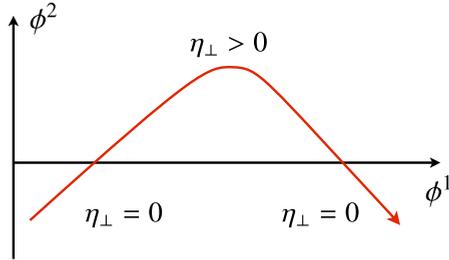
Physically, this result may be interpreted as coming from the fact that heavy and light modes are interchanging energy at a constant rate, therefore rendering only a change in the overall amplitude of the spectrum. However, as manifest from the effective theory (eq. 5.38), the speed of sound is modified as

$$c_s^2 = e^{-\beta} = \left(1 + 4\eta_{\perp}^2 \frac{H^2}{M^2}\right)^{-1}. \quad (5.72)$$

This implies the generation of nongaussianity noticeable in the bispectrum, as studied in Chen and Wang (2010a,b).

### 5.4.2 Single turn in the trajectory

As a next step, we consider the presence of a single turn in the inflationary trajectory. To simplify our analysis, we consider the specific case in which the trajectory is initially autoparallel to a geodesic (a straight path), then goes through a short period in which it suffers a turn, and finally goes back to the curve autoparallel to a geodesic. Figure 5.2 shows a prototype example of such a situation. We also assume



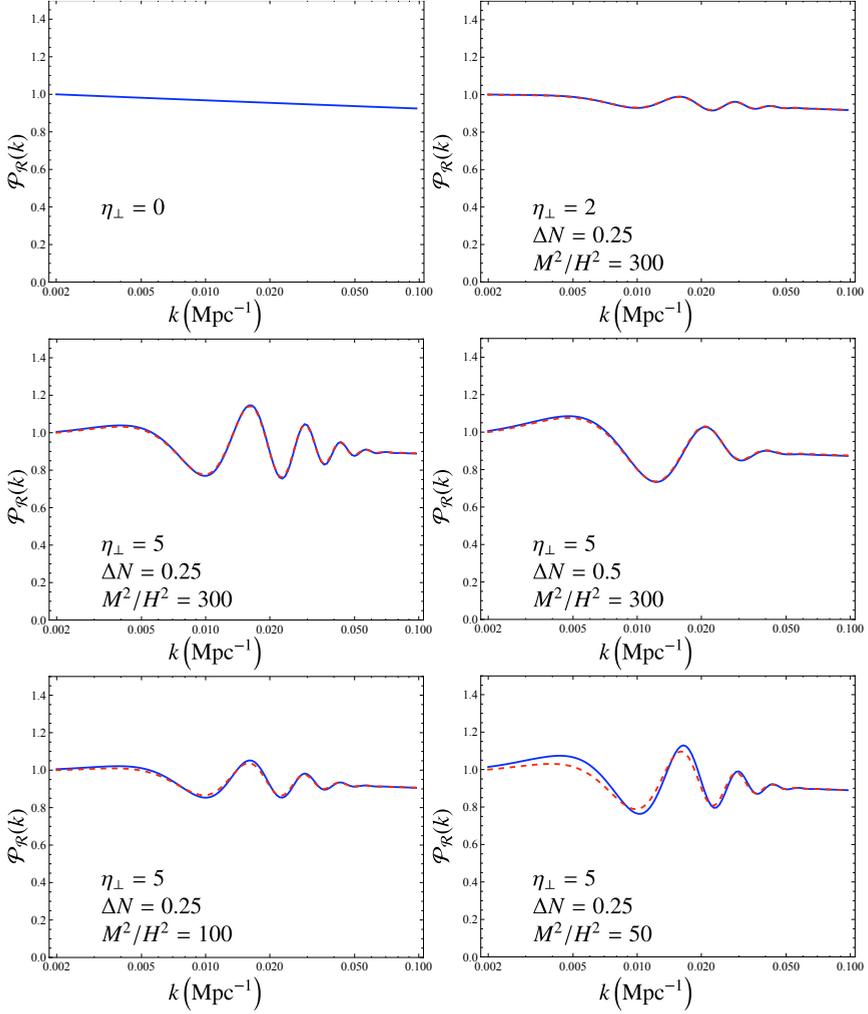
**Figure 5.2:** The figure shows a prototype example of a trajectory which suffers a localised bend towards the right.

that throughout this process all the slow-roll parameters except for  $\eta_{\perp}$  remain nearly constant. To model this situation, we take  $\eta_{\perp}$  to be an analytical function of the  $e$ -fold number  $N$  in the following way:

$$\eta_{\perp}(N) = \frac{\eta_{\perp\max}}{\cosh^2 [2(N - N_0)/\Delta N]}, \quad (5.73)$$

where  $\Delta N$  is the number of  $e$ -folds during which the bending happens, and  $N_0$  is the  $e$ -fold value at which the bending is at its peak, in which case  $\eta_{\perp}(N_0) = \eta_{\perp\max}$ . We recall that  $N$  may be suitably defined from conformal time  $\tau$  through the relation  $dN = aHd\tau$ . For the other slow-roll parameters we choose the reference values  $\epsilon = 0.022$  and  $\eta_{\parallel} = 0.034$ . These values correspond to a spectral index  $n_{\mathcal{R}} = 0.98$  and to a tensor to scalar ratio  $r = 0.35$ , which are marginally compatible with current CMB tests (Larson et al., 2010). Additionally, these values imply  $H = 10^{-5} M_{\text{Pl}}$ . Figure 5.3 shows the power spectra for six cases with different choices of the parameters  $\Delta N$ ,  $\eta_{\perp\max}$  and  $M^2$ . The plots contain both the spectrum obtained by solving the full coupled system of equations (solid line) and the spectrum obtained by solving the effective single-field equation of motion (dashed line). For simplicity, we normalise our results in units of  $2.46 \times 10^{-9}$  and give the scale  $k$  in units of  $\text{Mpc}^{-1}$ . As a reference, we have included the case  $\eta_{\perp} = 0$ , which corresponds to the power spectrum that would be obtained in the single-field case.

The main characteristic shown by the plots are oscillatory features appearing in the spectrum. It may be noticed that the  $e$ -fold width  $\Delta N$  during which the turn takes place actually set the scale  $k$  of the oscillatory features. On the other hand, the amplitude of the oscillations is roughly dictated by the ratio  $4\eta_{\perp\max}H^2/M^2$ . More precisely, the amplitude of the largest oscillatory feature is of order  $\delta\mathcal{P}_{\mathcal{R}}/\mathcal{P}_{\mathcal{R}} \sim 4\eta_{\perp\max}H^2/M^2$ , which agrees with the result of (eq. 5.70). Additionally, the match between the curve



**Figure 5.3:** The primordial power spectrum  $\mathcal{P}_{\mathcal{R}}(k)$  normalised in units of  $2.46 \times 10^{-9}$ , obtained for six different choices of  $\Delta N$ ,  $\eta_{\perp, \text{max}}$  and  $M^2$ . The plots show a comparison between the power spectrum obtained using the full system of equations (solid line) and the one obtained using the effective theory (dashed line). We have chosen as a pivot scale the value  $k_* = 0.002 \text{Mpc}^{-1}$ .

predicted by the effective theory and the full set of equations becomes better as  $M^2/H^2$  acquires larger values, irrespective of how large is  $\beta$ . In fact, in all of the examples shown we have  $\beta \sim 1$ .

The appearance of oscillatory features, not just a single bump, in the spectrum reflects the fact that both modes  $v^N$  and  $v^T$  backreact at sub-horizon scales as the turn happens. Once both modes cross the horizon, the amplitude of the adiabatic mode becomes frozen (therefore capturing the moment in which the mode was receiving or releasing energy) while the amplitude of the heavy mode quickly decays due to the accelerated expansion. In fact, we have checked that the levels of isocurvature perturbations at the end of inflation are negligible.

### 5.4.3 A specific example

As a last step towards understanding the effects of curved trajectories, we discuss our results applied to a specific toy model, where turns are produced due to the non-trivial evolution of the sigma model metric. Let us consider a two-field model with fields  $\phi^1 = \chi$  and  $\phi^2 = \psi$  with a kinetic term containing the following sigma model metric:

$$\gamma_{ab} = \begin{pmatrix} 1 & \Gamma(\chi) \\ \Gamma(\chi) & 1 \end{pmatrix}, \quad (5.74)$$

where  $\Gamma(\chi)$  is only a function of the  $\chi$  field and restricted to satisfy  $\Gamma^2(\chi) < 1$ . The non-vanishing connections are  $\Gamma_{\chi\chi}^\chi = -\Gamma_\chi/(1 - \Gamma^2)$  and  $\Gamma_{\chi\chi}^\psi = \Gamma_\chi/(1 - \Gamma^2)$  with  $\Gamma_\chi = \partial_\chi \Gamma$ , and the equations of motion for the background fields are found to be

$$\ddot{\chi} - \frac{\Gamma\Gamma_\chi}{1 - \Gamma^2} \dot{\chi}^2 + 3H\dot{\chi} + \frac{1}{1 - \Gamma^2} V_\chi - \frac{\Gamma}{1 - \Gamma^2} V_\psi = 0, \quad (5.75)$$

$$\ddot{\psi} + \frac{\Gamma_\chi}{1 - \Gamma^2} \dot{\chi}^2 + 3H\dot{\psi} + \frac{1}{1 - \Gamma^2} V_\psi - \frac{\Gamma}{1 - \Gamma^2} V_\chi = 0, \quad (5.76)$$

where  $V_\chi = \partial_\chi V$  and  $V_\psi = \partial_\psi V$ . For concreteness, let us consider the following separable scalar field potential:

$$V(\chi, \psi) = V_0(\chi) + \frac{1}{2} M^2 \psi^2. \quad (5.77)$$

In the particular case of  $\Gamma = 0$ , the dynamics of the two fields decouple and inflation may be achieved with  $\chi$  by a suitable choice of the potential  $V_0(\chi)$ . If, however,  $\Gamma(\chi)$  is allowed to be non-vanishing for certain values of  $\chi$ , then a mixing between the two modes is inevitable, and the inflationary trajectory will be curved. Following

the discussion at the beginning of Section 5.2, we choose the tangential and normal vectors  $T^a$  and  $N^a$  as in (eq. 5.4) and (eq. 5.5):

$$T^a = \frac{1}{\dot{\phi}_0} (\dot{\chi}, \dot{\psi}), \quad (5.78)$$

$$N^a = \frac{1}{\dot{\phi}_0 \sqrt{1 - \Gamma^2}} (-\dot{\psi} - \Gamma \dot{\chi}, \dot{\chi} + \Gamma \dot{\psi}), \quad (5.79)$$

where  $\dot{\phi}_0 = \dot{\chi}^2 + \dot{\psi}^2 + 2\Gamma \dot{\chi} \dot{\psi}$ . Recall that with this convention  $\eta_\perp$  is allowed to change its sign. The relevant background parameters describing this situation are then

$$\epsilon = \frac{\dot{\chi}^2 + \dot{\psi}^2 + 2\Gamma \dot{\chi} \dot{\psi}}{2M_{\text{pl}}^2 H^2}, \quad (5.80)$$

$$\eta_\parallel = 3 + \frac{\dot{\chi} V_\chi + \dot{\psi} V_\psi}{H (\dot{\chi}^2 + \dot{\psi}^2 + 2\Gamma \dot{\chi} \dot{\psi})}, \quad (5.81)$$

$$\eta_\perp = - \frac{(\dot{\psi} + \Gamma \dot{\chi}) V_\chi - (\dot{\chi} + \Gamma \dot{\psi}) V_\psi}{H \sqrt{1 - \Gamma^2} (\dot{\chi}^2 + \dot{\psi}^2 + 2\Gamma \dot{\chi} \dot{\psi})}, \quad (5.82)$$

where  $H$  is given by  $6M_{\text{pl}}^2 H^2 = \dot{\chi}^2 + \dot{\psi}^2 + 2\Gamma \dot{\chi} \dot{\psi} + 2V$ . For concreteness, let us consider a parameter  $\Gamma(\chi)$  having the following  $\chi$ -dependence:

$$\Gamma(\chi) = \frac{\Gamma_0}{\cosh^2 [2(\chi - \chi_0)/\Delta\chi]}, \quad (5.83)$$

where  $\Gamma_0$  is the maximum value attained by  $\Gamma(\chi)$ . We take the potential  $V_0(\chi)$  as

$$V_0(\chi) = \frac{1}{256V_0^5} \left( 16V_0^3 + V_1^2 \chi^3 - 2V_0 V_1 \chi^2 (V_1 + 2V_2 \chi) + 8V_0^2 \chi (V_1 + \chi(V_2 + V_3 \chi)) \right)^2, \quad (5.84)$$

with  $V_0 = 3H_i$ ,  $V_1 = -\sqrt{2\epsilon_i} V_0$ ,  $V_2 = V_0(\epsilon_i + \eta_i)$  and  $V_3 = 10^{-4} V_0$ , where, as before,  $H_i$ ,  $\epsilon_i$  and  $\eta_i$  render values  $\epsilon = 0.022$  and  $\eta_\parallel = 0.034$  for the slow-roll parameters in the absence of curves. For this specific configuration, we found that the background value of  $\epsilon(\tau)$  remains nearly constant at the attractor value  $\epsilon = 0.022$  whereas the background value of  $\eta_\parallel(\tau)$  is more sensitive to the turns suffered by the trajectory, having small deviations from the attractor value  $\eta_\parallel = 0.034$ . Additionally, we found two relevant time scales determining the behaviour of background quantities  $\eta_\parallel$  and  $\eta_\perp$ :

$$T_\psi \equiv M^{-1}, \quad (5.85)$$

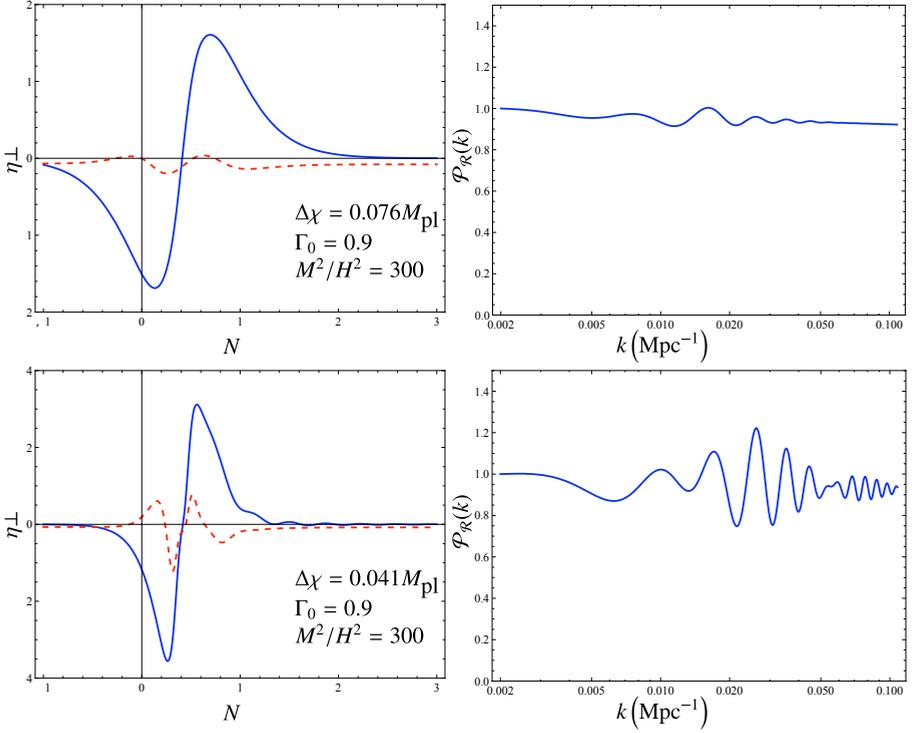
$$T_\chi \equiv \frac{\Delta\chi}{\dot{\phi}_0} = \frac{\Delta\chi}{\sqrt{2\epsilon} M_{\text{pl}} H}. \quad (5.86)$$

The appearance of these time scales are actually easy to understand. First, notice that  $T_\chi$  is the time during which the turn takes place whereas  $T_\psi$  is the oscillation period of the massive field  $\psi$ . We find that if  $T_\chi \ll T_\psi$ , then the background dynamics is such that  $\phi_0^a = (\chi, \psi)$  oscillates about  $\psi = 0$ , meaning that both  $\eta_{\parallel}$  and  $\eta_{\perp}$  presented oscillatory features with frequency  $\mathcal{O}(T_\psi^{-1})$ . On the other hand, if  $T_\chi \gg T_\psi$ , the background field departs adiabatically from the minima of the potential  $\psi = 0$ , and the time evolution of  $\eta_{\parallel}$  and  $\eta_{\perp}$  is dictated by the time scale  $T_\chi$ . This latter case may be interpreted as a situation where the trajectory is momentarily pushed towards one of the walls of the potential, as the curve takes place. Figure 5.4 shows the background values of  $\eta_{\perp}$  and  $\eta_{\parallel}$  (as functions of the  $e$ -fold number  $N$ ) for the case  $\Gamma_0 = 0.9$ ,  $M^2 = 300H^2$  and two values of  $\Delta\chi$ , namely  $\Delta\chi = 0.076M_{\text{Pl}}$  and  $\Delta\chi = 0.041M_{\text{Pl}}$ . In the latter case, it may be appreciated how the time scale  $T_\psi$  appears mildly in the shape of  $\eta_{\perp}$ .

The figure also shows the power spectrum obtained for the two described cases (right panels). In the present examples, the features appearing in the spectrum are not as regular as those of Figure 5.3. This is mainly because in the present situation the curvilinear trajectory contains several turns, in order to go back to the attractor solution. Although in this specific model the slow-roll parameter  $\eta_{\parallel}$  appears to be sensitive to the mass scale  $M$  and the curves taking place, it is important to notice that this is a model dependent characteristic, and that in general  $\eta_{\parallel}$  may show various types of behaviour depending on the sigma model metric and the potential. In general, however, the momentary time variation of  $\eta_{\parallel}$  due to curved trajectories does not spoil the slow-roll regime, and background fields tend to quickly evolve back to the attractor behaviour characteristic of the single-field case as soon as the bending of the trajectory stops. In this regard, we find that the time variation of  $\eta_{\parallel}$  is not relevant for the appearance of features in the power spectrum, and that the main contribution is coming from the derivative interactions due to  $\eta_{\perp}$  in the equations of motion.

#### 5.4.4 Enhancement of nongaussianity

We briefly elaborate here on another potentially observable feature which is so far not discussed. In the previous section the power spectrum of the curvature perturbation was computed for a few examples where the inflaton traverses sufficiently curved regions in field space. From the results, it is clear that features in the spectrum will be generated each time the trajectory traverses a bend. These features are produced via the kinetic interaction between the heavy isocurvature modes and the light curvature mode as the turns are traversed by the background field. Crucially, in these examples the heavy mode remained very massive throughout ( $M^2 \gg H^2$ ), highlighting the



**Figure 5.4:** Left panels: the evolution of  $\eta_{\perp}$  (solid line) and  $10 \times \eta_{\parallel}$  (dashed line) as functions of e-fold number  $N$  for two set of values of parameters  $\Delta\chi$ ,  $\Gamma_0$  and  $M^2/H^2$ . In the first case,  $\Delta\chi = 0.076 M_{\text{pl}}$  and the maximum value of  $\eta_{\perp}$  is about  $|\eta_{\perp}| \approx 1.7$ , whereas in the second case,  $\Delta\chi = 0.041 M_{\text{pl}}$  and the maximum value becomes  $|\eta_{\perp}| \approx 3.5$ . Right panels: the resulting primordial power spectrum  $\mathcal{P}_{\mathcal{R}}(k)$ , normalised in units of  $2.46 \times 10^{-9}$ , obtained for the set of parameters used in the plots of  $\eta_{\perp}$ . The scale  $k$  appears in units of  $\text{Mpc}^{-1}$ .

fact that heavy fields may not always be disregarded (truncated) when computing the spectrum for adiabatic modes.

What is important to note is that the interaction between curvature and isocurvature modes implies a change in the speed of sound for the curvature perturbations – as long as  $M^2 \gg H^2$ ,  $\beta(\tau, k)$  is effectively  $k$ -independent before horizon crossing and

the speed of sound may be written as

$$c_s^2 = e^{-\beta} = \left(1 + 4\eta_{\perp}^2 \frac{H^2}{M^2}\right)^{-1}. \quad (5.87)$$

As is well known, a model with a speed of sound significantly smaller than unity gives rise to a noticeable level of nongaussianity of equilateral type, characterised by the non-linear parameter (Bartolo et al., 2004)

$$f_{\text{NL}}^{(\text{eq})} \sim \frac{1}{c_s^2}. \quad (5.88)$$

Thus, we are led to reason that for generic models of inflation with curvilinear trajectories in a multi-dimensional field space, glitches in the power spectrum are accompanied by a correlated enhancement of nongaussianity of the equilateral type, provided that the turns in the inflaton trajectory violate the adiabatic approximation vigorously enough – a phenomenon which we have argued occurs at various points in field space in many realistic realisations of inflation. Thus although there appear to be many models where either non-trivial modulations in the power spectrum (e.g. features in the single-field inflaton potential, Starobinsky, 1992, Adams et al., 2001, Tocchini-Valentini et al., 2005, Gong, 2005, Covi et al., 2006, Hunt and Sarkar, 2007, Ichiki et al., 2010, Peiris and Verde, 2010, Hamann et al., 2010) or large equilateral nongaussianity (e.g. DBI inflation, Silverstein and Tong, 2004, Alishahiha et al., 2004) result, it appears that in generic multi-field models with curved inflationary trajectories, both are present and correlated. Evidently, the effective quadratic action (eq. 5.41) contains the leading higher order corrections which can also result in non-Gaussian signatures and implies the non-linear parameter (eq. 5.88) (Cheung et al., 2008). However, to fully describe the bispectrum associated with the curvature perturbation, we need to properly take into account the cubic order action including gravity. We will discuss this issue in a separate publication.

## 5.5 Conclusions

Multi-field models of inflation contain a range of physics which goes beyond that encountered within the single-field paradigm. In this work we have focused on the particular case where all of the scalar fields remain massive during inflation except for one, which slowly rolls down the multi-field potential. We have found that curved inflationary trajectories can generate significant features in the primordial spectrum of

density perturbations arising from normal modes becoming excited and backreacting on the dynamics of the adiabatic mode.

To achieve these results, an extension of Groot Nibbelink and van Tent (2000, 2002), we analysed the evolution of the quantum perturbations of a general multi-field setup, including the presence of a non-canonical kinetic term. Our methods are completely general and naturally incorporate those implemented in previous works (Lalak et al., 2007b, Tsujikawa et al., 2003), where stochastic Gaussian variables are used. Moreover, although the main focus of this work was the study of systems where there exists a hierarchy, our results may be used to study a wide range of situations, including situations where no such hierarchies are present.

Our formalism allows us to consider time-dependent situations beyond the regime of applicability of existing methods, such as inflaton trajectories with fast, sudden turns (regardless of whether the sigma model metric is canonical or non-canonical) as well as more general situations in which the masses of the heavy fields in the orthogonal direction are changing along the trajectory (even if they still remain much heavier than  $H^2$  and all other scales of interest). Additionally, we wish to emphasise that these non-decoupling effects have their origin in the non-geodesic nature of the trajectories in field space.<sup>7</sup>

Our results highlight the limitations of simply truncating heavy physics when modelling single-field realisations of inflation and show under which circumstances high energy effects can leave an imprint on the power spectrum. The main reason behind these effects is the existence of kinetic couplings between adiabatic and non-adiabatic modes, emerging as the inflationary trajectory suffers a turn. As we have seen in section 4.3.1, it is always possible to change basis to a canonical frame where such interactions are absent. In that case, the eigenvectors of the perturbation mass matrix quickly vary as the inflationary trajectory turns, and we are left with the alternative point of view by which these high energy effects appear due to a violation of the adiabatic condition for truncating heavy fields. In fact, if the heavy fields are sufficiently massive, we find that we can construct an effective field theory for the adiabatic modes encapsulating the relevant effects of the full multi-field dynamics. As we have seen, such effects are not mere corrections to the standard single-field theory, but represent entirely new contributions to the quadratic action for perturbations.

Particularly noteworthy is the presence of potentially observable signatures that result from a reduced speed of sound for the adiabatic perturbations during sudden turns. As a corollary, correlated nongaussianity will also become manifest as a result of these sudden turns although a full analysis studying the details of their appearance

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<sup>7</sup>Recent work by Cremonini et al. (2010b) discusses some of these effects in a particular model, the so-called gelaton model of Tolley and Wyman (2010).

## **Chapter 5: Two-field models of inflation**

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in multi-field inflation is beyond the scope of this chapter and will be addressed in a future report. Nevertheless, it would appear that in generic multi-field models with curved inflationary trajectories, both effects are present and correlated, and can potentially give information about other, much heavier, fields that would otherwise be inaccessible to experiment.

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## CHAPTER 6

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# Conclusions

In studying low energy physics, it is usually assumed that the high energy physics is decoupled and does not affect the low energy physics. Even though gravity is always there, gravitational corrections give rise to Planck suppressed operators, its effects are usually assumed to be negligible. This assumption seems reasonable at first sight. The Planck scale is huge compared to any scale that is relevant for almost the complete history of the universe, including the inflationary epoch. Planck suppressed operators will thus be heavily suppressed for all relevant physics.

However, in the research presented in this thesis it is shown that this conclusion is drawn too hastily. As shown in chapter 2, decoupling is non-trivial in supergravity, the only consistent framework to study gravitational effects in field theory. Sectors that do not decouple participate in the dynamics and, therefore, strongly suppressed operators can still have a profound effect by changing the vacuum structure.

In chapter 3 a case study of a supergravity theory is presented, where a vacuum energy expectation value contribution is generated by a gravitational minimally coupled sector that breaks supersymmetry. It is shown that, along with generating this nonzero VEV, the coupling has the effect of changing the stability properties of the extrema of the potential. Local minima will be destabilised for an arbitrary amount of uplifting. Similar in spirit, local maxima — that are stable in Anti-de Sitter due to the Breitenlohner-Freedman bound (eq. 1.53) — can change the sign of its second derivative and become stable after uplifting. Although these vacua seem problematic from a phenomenological point of view, they are very interesting from a conceptual point of view.

Chapters 4 and 5 deal in detail with the case of non-decoupled sectors. Instead of supergravity, the derivation is done in the more general framework of the nonlinear sigma model. In chapter 4 the framework for calculating the properties of a field evolving along a trajectory in field space is introduced. The trajectory is calculated, and it is shown that for general trajectories the adiabatic approximation does not hold. The general equations for the perturbation theory are calculated. Then, the behaviour in the Minkowski limit is studied, and it is shown that the requirement on a Kähler function derived in chapter 2 is indeed the sufficient condition for a sector to decouple.

In chapter 5 this framework is applied to inflation. As the relevant physics is already evident in the minimal extension of a two-field model, this model is presented. It is shown that it is possible, in the case of a large mass hierarchy between the fields, to obtain an effective single-field theory. However, this single field theory is characterised by a speed of sound that is less than the speed of light, with its value determined by the turn rate divided by the mass of the heavy direction. It is shown that this effective field theory agrees, in the large hierarchy limit, with the two-field calculation. The effect of this time dependent speed of sound is that the power spectrum develops a scale dependence, as oscillations in the power spectrum are formed when the speed of sound changes. Furthermore, a reduced speed of sound is known to produce equilateral nongaussianities.

What remains to be studied are non-geodesic trajectories in supergravity. As discussed in chapter 4, basically all supergravity models of the form

$$G(H, \bar{H}, L, \bar{L}) = f_1(L, \bar{L}) + (H - H_0)^2 f_2(H, \bar{H}, L, \bar{L}) + (\bar{H} - \bar{H}_0)^2 f_3(H, \bar{H}, L, \bar{L}) + (H - H_0)(\bar{H} - \bar{H}_0) f_4(H, \bar{H}, L, \bar{L}) \quad (6.1)$$

for arbitrary functions  $f_1$ ,  $f_2$ ,  $f_3$  and  $f_4$  can be consistently decoupled along curves of  $H = H_0$ , where  $H_0$  is an extremum of the  $H$ -fields, and thus allow for geodesic trajectory. However, a parameter  $\Delta$  was introduced, showing that often the evolution of fields is such that fields evolve *not* along minima of the potential. It is exactly this reason that causes the KKLT model to receive large corrections, as calculated in the conclusion of chapter 2: minima in the truncated theory are *not* minima in the full theory. Thus, consistent decoupling fails because using the truncated theory causes one to find the wrong minimum. In chapter 5 it is shown that properly integrating out fields can lead to an effective theory that differs significantly from a theory obtained by naïve truncation. These differences in supergravity language would relate  $\Delta$ , the deviation from the minimum, and thus  $\beta^{-1}$ , the speed of sound of the effective theory, to the parameter  $\delta$  of (eq. 2.26), which signifies the difference between the proper calculation of the supergravity minimum and the supergravity minimum that one obtains from an effective theory obtained by truncating the heavy physics.

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To conclude, non-decoupling of heavy scalars is a general phenomenon in the physics of evolving vacua. Therefore, one cannot expect heavy physics to be decoupled during the cosmological evolution. Since these effects do not decouple, cosmology — and in particular inflation — is a powerful probe to study these heavy degrees of freedom. It can be expected that these heavy degrees of freedom left signatures in the power spectrum and bispectrum. In this light, the improved measurement of the Planck mission will be very relevant.



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## APPENDIX A

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# Commutation relations for quantum multi-fields

In this appendix we show that the commutation relations (eq. 4.61) in section 4.3.2 are fully consistent with the evolution of the  $v_\alpha^J(k, \tau)$  dictated by the set of equations of motion (eq. 4.64). To begin with, observe that in order to satisfy the commutation relation (eq. 4.61) the  $N$  mode solutions  $v_\alpha^J(k, \tau)$  must satisfy the following conditions:

$$\sum_\alpha \left[ v_\alpha^J \frac{\mathcal{D}v_\alpha^{J*}}{d\tau} - \frac{\mathcal{D}v_\alpha^J}{d\tau} v_\alpha^{J*} \right] = i\delta^{JJ}, \quad (\text{A.1})$$

$$\sum_\alpha \left[ v_\alpha^J v_\alpha^{J*} - v_\alpha^{J*} v_\alpha^J \right] = 0, \quad (\text{A.2})$$

$$\sum_\alpha \left[ \frac{\mathcal{D}v_\alpha^J}{d\tau} \frac{\mathcal{D}v_\alpha^{J*}}{d\tau} - \frac{\mathcal{D}v_\alpha^J}{d\tau} \frac{\mathcal{D}v_\alpha^{J*}}{d\tau} \right] = 0. \quad (\text{A.3})$$

## Appendix A: Commutation relations for quantum multi-fields

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To show that these relations are satisfied at any given time  $t$  we proceed as follows: first, let us define the tensors

$$A^{IJ} = i \sum_{\alpha} \left[ v_{\alpha}^I v_{\alpha}^{J*} - v_{\alpha}^J v_{\alpha}^{I*} \right], \quad (\text{A.4})$$

$$B^{IJ} = i \sum_{\alpha} \left[ \frac{\mathcal{D}v_{\alpha}^I}{d\tau} \frac{\mathcal{D}v_{\alpha}^{J*}}{d\tau} - \frac{\mathcal{D}v_{\alpha}^J}{d\tau} \frac{\mathcal{D}v_{\alpha}^{I*}}{d\tau} \right], \quad (\text{A.5})$$

$$E^{IJ} = i \sum_{\alpha} \left[ v_{\alpha}^I \frac{\mathcal{D}v_{\alpha}^{J*}}{d\tau} - \frac{\mathcal{D}v_{\alpha}^J}{d\tau} v_{\alpha}^{I*} \right]. \quad (\text{A.6})$$

These tensors satisfy the properties

$$A^{IJ} = A^{IJ*} = -A^{JI}, \quad (\text{A.7})$$

$$B^{IJ} = B^{IJ*} = -B^{JI}, \quad (\text{A.8})$$

$$E^{IJ} = E^{IJ*}. \quad (\text{A.9})$$

In other words, they are real, with  $A^{IJ}$  and  $B^{IJ}$  antisymmetric while  $E^{IJ}$  has no specific symmetries. Because of these properties  $A^{IJ}$  and  $B^{IJ}$  consist of  $\mathcal{N}(\mathcal{N}-1)/2$  independent real components each, whereas  $E^{IJ}$  consists of  $\mathcal{N}^2$  independent real components. Thus, in order to fix the values of all of these tensors we need to specify  $2\mathcal{N}^2 - \mathcal{N}$  independent quantities. These tensors also satisfy the following equations of motion:

$$\frac{\mathcal{D}}{d\tau} A^{IJ} = E^{IJ} - E^{JI}, \quad (\text{A.10})$$

$$\frac{\mathcal{D}}{d\tau} B^{IJ} = \Omega^I_{\ K} E^{KJ} - \Omega^J_{\ K} E^{KI}, \quad (\text{A.11})$$

$$\frac{\mathcal{D}}{d\tau} E^{IJ} = B^{IJ} + A^{IK} \left( k^2 \delta_K^J + \Omega_K^J \right). \quad (\text{A.12})$$

Taking the trace to the last equation, we obtain that the trace  $E \equiv E^I_{\ I}$  satisfies

$$\frac{dE}{d\tau} = 0, \quad (\text{A.13})$$

and therefore  $E$  is a constant of motion of the system. Furthermore, observe that the configuration  $E^{IJ} = E\delta^{IJ}/\mathcal{N}$  and  $A^{IJ} = B^{IJ} = 0$  for which conditions (eq. A.1-A.3) are satisfied corresponds to a fixed point of the set of equations (eq. A.10-A.12). That is, they automatically satisfy

$$\frac{\mathcal{D}}{d\tau} A^{IJ} = \frac{\mathcal{D}}{d\tau} B^{IJ} = \frac{\mathcal{D}}{d\tau} E^{IJ} = 0. \quad (\text{A.14})$$

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Therefore, it only remains to verify whether there exist sufficient independent degrees of freedom in order to satisfy the initial conditions  $E^{IJ} = E\delta^{IJ}/\mathcal{N}$  and  $A^{IJ} = B^{IJ} = 0$  at a given initial time  $\tau_i$ . As a matter of fact, we have exactly the right number of degrees of freedom. As we have already noticed there exists  $\mathcal{N}$  independent solutions  $v_\alpha^I(k, \tau)$  to the equations of motion. To fix each solution  $v_\alpha^I(k, \tau)$  we therefore need to specify  $2\mathcal{N}^2$  independent quantities, corresponding to the addition of  $\mathcal{N}^2$  components  $v_\alpha^I(\tau_i)$  and  $\mathcal{N}^2$  momenta  $\mathcal{D}v_\alpha^I/d\tau(\tau_i)$ . However, we must notice that the overall phase of each solution  $v_\alpha^I(k, \tau)$  plays no roll in setting the initial values for  $A^{IJ}$ ,  $B^{IJ}$  and  $E^{IJ}$ . We therefore have precisely  $2\mathcal{N}^2 - \mathcal{N}$  free parameters to set  $E^{IJ} = E\delta^{IJ}/\mathcal{N}$  and  $A^{IJ} = B^{IJ} = 0$ . Of course, the value of the trace of  $E$  is part of this freedom, and we are free to fix it in such a way that  $E/\mathcal{N} = 1$ .

To summarise, it is always possible to choose the initial conditions for  $v_\alpha^I(k, \tau)$  and  $\mathcal{D}v_\alpha^I/d\tau(k, \tau)$  in such a way that conditions (eq. A.1-A.3) are satisfied. These conditions ensure the commutation relation (eq. 4.61). To finish this discussion, recall that one possible choice for the initial conditions for the perturbations allowing (eq. A.1) to (eq. A.2) to be satisfied, are precisely those expressed in (eq. 4.67), with suitable choices for the coefficients  $v_\alpha(k)$  and  $\pi_\alpha(k)$ :

$$v_\alpha(k)\pi_\alpha^*(k) - v_\alpha^*(k)\pi_\alpha(k) = i, \quad (\text{A.15})$$

for  $\alpha = 1, \dots, \mathcal{N}$ . We should emphasise however that this is not the unique choice for initial conditions and, in general, any choice for which  $E^{IJ} = E\delta^{IJ}/\mathcal{N}$  and  $A^{IJ} = B^{IJ} = 0$  will do just fine.



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## APPENDIX B

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# Zeroth-order theory of the background fields

In this appendix we study in detail the dynamics offered by the tree level potential  $V(\phi) = V_*(\phi)$  discussed in Section 4.4.1. We shall focus only on potentials  $V$  for which the Hessian  $V_{ab}$  is positive definite. Let us for a moment independently consider solutions to the equation

$$V^a = 0. \tag{B.1}$$

In general, these will correspond to a set of fields parametrising a surface  $\mathcal{S}$  in  $\mathcal{M}$ . The fields lying on this surface correspond to exactly flat directions of the potential  $V$ . Let us express this surface by means of the parametrisation

$$\phi_*^a = \phi_*^a(\chi^\alpha), \tag{B.2}$$

where  $\alpha = 1, \dots, n_S$ , with  $n_S$  the number of flat directions of the potential. Then

$$V_a[\phi_*(\chi)] = 0 \tag{B.3}$$

for any  $\chi$ . Clearly,  $n_S$  is the dimension of the surface. We may now define the induced metric on the surface by making use of the pullbacks  $X^a_\alpha \equiv \partial_\alpha \phi_*^a$ :

$$g_{\alpha\beta} = X^a_\alpha X^b_\beta \gamma_{ab}. \tag{B.4}$$

Let us for a moment disregard the degrees of freedom perpendicular to this surface and consider only those lying on  $\mathcal{S}$ . This corresponds to truncating the theory by

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considering only the fields  $\chi^\alpha$ . The theory for such fields would be deduced from the action

$$S = -\frac{1}{2} \int d^4x g_{\alpha\beta} \partial_\mu \chi^\alpha \partial^\mu \chi^\beta, \quad (\text{B.5})$$

and the equations of motion would be given by

$$\frac{D}{dt} \dot{\chi}^\alpha = \frac{d^2 \chi^\alpha}{dt^2} + \hat{\Gamma}_{\beta\gamma}^\alpha \frac{d\chi^\beta}{dt} \frac{d\chi^\gamma}{dt} = 0, \quad (\text{B.6})$$

where

$$\hat{\Gamma}_{\beta\gamma}^\alpha = \frac{1}{2} g^{\alpha\delta} (\partial_\beta g_{\delta\gamma} + \partial_\gamma g_{\beta\delta} - \partial_\delta g_{\beta\gamma}) \quad (\text{B.7})$$

is the connection deduced out of the induced metric  $g_{\alpha\beta}$ . The relation between  $\hat{\Gamma}_{\beta\gamma}^\alpha$  and  $\Gamma_{bc}^a$  is given by

$$\hat{\Gamma}_{\beta\gamma}^\alpha = X_a^\alpha (X_\beta^b X_\gamma^c \Gamma_{bc}^a + X_{\beta\gamma}^a), \quad (\text{B.8})$$

where  $X_{\beta\gamma}^a \equiv \partial_\gamma X_\beta^a$ . It is convenient here to define  $M_{\beta\gamma}^a \equiv X_\beta^b X_\gamma^c \Gamma_{bc}^a + X_{\beta\gamma}^a$ , which yields  $\hat{\Gamma}_{\beta\gamma}^\alpha = X_a^\alpha M_{\beta\gamma}^a$ . To review under what conditions the previous truncation is consistent, let us recall how much a solution to (eq. B.6) deviates from the equation of motion of the full theory given by (eq. 4.6). By differentiating with respect to time the solution (eq. B.2) with  $\chi^\alpha$  satisfying (eq. B.6), we find

$$\frac{D}{dt} \dot{\phi}_*^a(\chi) = X_a^\alpha \ddot{\chi}^\alpha + M_{\alpha\beta}^a \dot{\chi}^\alpha \dot{\chi}^\beta \quad (\text{B.9})$$

$$\implies \frac{D}{dt} \dot{\phi}_*^a(\chi) = (M_{\alpha\beta}^a - X_\gamma^a X_b^\gamma M_{\alpha\beta}^b) \dot{\chi}^\alpha \dot{\chi}^\beta. \quad (\text{B.10})$$

It is useful to define  $Q_{\alpha\beta}^a \equiv P_b^a M_{\alpha\beta}^b$ , where  $P_b^a \equiv \delta_b^a - X_\gamma^a X_b^\gamma$  is the projector along the space perpendicular to the surface.  $Q_{\alpha\beta}^a$  transforms as a tensor:

$$Q_{\alpha\beta}^a = \partial_\alpha X_\beta^a + \Gamma_{b\alpha}^a X_\beta^b - \hat{\Gamma}_{\alpha\beta}^\gamma X_\gamma^a = D_\alpha X_\beta^a, \quad (\text{B.11})$$

where  $\Gamma_{b\alpha}^a \equiv \Gamma_{bc}^a X_\alpha^c$ . The previous notation is consistent as  $X_\alpha^a$  transforms homogeneously under reparametrisations of  $\phi$  and  $\chi$ . Thus, finally we are left with

$$\frac{D}{dt} \dot{\phi}_*^a(\chi) = Q_{\alpha\beta}^a \dot{\chi}^\alpha \dot{\chi}^\beta. \quad (\text{B.12})$$

Therefore, since  $V^a(\phi_*) = 0$  by definition, if  $Q_{\alpha\beta}^a \dot{\chi}^\alpha \dot{\chi}^\beta$  is non-vanishing along the trajectory followed by  $\chi^\alpha$ , then  $\phi_*^a$  does not satisfy the equations of motion for  $\phi^a$  in the full theory. In fact, since we are interested in an arbitrary solution  $\chi^\alpha = \chi^\alpha(t)$

of (eq. B.6), in general either  $\dot{\chi}^\alpha = 0$  or  $Q_{\alpha\beta}^a = 0$ . The first case corresponds to a stationary solution, where the background is not evolving. The second case  $Q_{\alpha\beta}^a = 0$  is more interesting, as it corresponds to the case in which  $\mathcal{S}$  is geodesically generated. To appreciate this, notice first that if  $Q_{\alpha\beta}^a = 0$  then  $\phi_*^a = \phi_*^a(t)$  satisfies the equation of a geodesic. In second place, it is possible to deduce the following identity

$$\begin{aligned}\mathcal{R}^a_{\alpha\beta\gamma} &\equiv P^a_b X^c_\alpha X^d_\beta X^e_\gamma \mathcal{R}^B_{cde} \\ &= P^a_b \left( D_\beta Q^b_{\gamma\alpha} - D_\gamma Q^b_{\beta\alpha} \right).\end{aligned}\quad (\text{B.13})$$

Thus, if  $Q_{\alpha\beta}^a = 0$  then arbitrary vectors, which are tangent to  $\mathcal{S}$ , will not generate a component normal to  $\mathcal{S}$  after being transported around an arbitrary loop in  $\mathcal{S}$ . Finally, one also has the general relation

$$\hat{\mathcal{R}}^\alpha_{\beta\gamma\delta} = X_a^\alpha X^b_\beta X^c_\gamma X^d_\delta \mathcal{R}^a_{bcd} + \left( Q^a_{\beta\delta} \gamma_{ab} Q^b_{\sigma\gamma} g^{\sigma\alpha} - Q^a_{\beta\gamma} \gamma_{ab} Q^b_{\sigma\delta} g^{\sigma\alpha} \right), \quad (\text{B.14})$$

meaning that if  $Q_{\alpha\beta}^a = 0$  one has that the Riemann tensor  $\hat{\mathcal{R}}^\alpha_{\beta\gamma\delta}$  characterising  $\mathcal{S}$  coincides with the induced Riemann tensor  $X_a^\alpha X^b_\beta X^c_\gamma X^d_\delta \mathcal{R}^a_{bcd}$  to the surface.

It is rather clear that whenever the surface  $\mathcal{S}$  is not geodesically generated, the solution  $\phi^A = \phi^A(\chi)$  is not a solution of the full set of equations of motion. Let us now ask under what circumstances this might be a good approximation. For this, consider the following notation for the full solution:

$$\phi^a = \phi_*^a + \Delta^a, \quad (\text{B.15})$$

where  $\Delta^a$  has the purpose of parametrising the displacement of the full solution from  $\phi_*^a$  defining the surface  $\mathcal{S}$ . To deduce the equation of motion for  $\Delta^a$  notice that

$$\begin{aligned}\frac{D\dot{\phi}^a}{dt} &= \ddot{\phi}^a + \Gamma^a_{bc}(\phi) \dot{\phi}^b \dot{\phi}^c \\ &= \ddot{\phi}_*^a + \ddot{\Delta}^a + \Gamma^a_{bc}(\phi_* + \Delta) \left( \dot{\phi}_*^b + \dot{\Delta}^b \right) \left( \dot{\phi}_*^c + \dot{\Delta}^c \right) \\ &= \frac{D\dot{\phi}_*^a}{dt} + \ddot{\Delta}^a + \Gamma^a_{bc}(\phi_*) \dot{\Delta}^b \dot{\phi}_*^c + \Gamma^a_{bc}(\phi_*) \dot{\phi}_*^b \dot{\Delta}^c + \partial_d \Gamma^a_{bc}(\phi_*) \dot{\phi}_*^b \dot{\phi}_*^c \Delta^d.\end{aligned}\quad (\text{B.16})$$

On the other hand, we have the relation

$$\frac{D^2\Delta^a}{dt^2} = \left[ \dot{\Delta}^a + \Gamma^a_{bc}(\phi_*) \Delta^b \dot{\phi}_*^c \right] \dot{\phi}_*^a + \Gamma^a_{bc}(\phi_*) \left[ \dot{\Delta}^b + \Gamma^b_{de}(\phi_*) \Delta^d \dot{\phi}_*^e \right] \dot{\phi}_*^c. \quad (\text{B.17})$$

Putting these two expressions together we find the equation of motion for  $\Delta^a$  to be given by

$$\frac{D^2\Delta^a}{dt} + Q^a_{\alpha\beta} \dot{\chi}^\alpha \dot{\chi}^\beta + C^a_b(\phi_*) \Delta^b = 0, \quad (\text{B.18})$$

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where we are neglecting terms of higher order in  $\Delta$ . In the previous expression we have defined

$$C^a_b(\phi_*) \equiv V^a_b(\phi_*) - \mathcal{R}^a_{cdb}(\phi_*)\dot{\phi}_*^c\dot{\phi}_*^d, \quad (\text{B.19})$$

where  $V^a_b(\phi_*) \equiv \gamma^{ac}(\phi_*)\nabla_c V_b(\phi_*)$ . In deriving this expression we have assumed that  $Q^a_{\alpha\beta}\dot{\chi}^\alpha\dot{\chi}^\beta$  is of  $\mathcal{O}(\Delta)$ . This is correct since we need to demand  $\Delta = 0$  for the particular case  $Q^a_{\alpha\beta}\dot{\chi}^\alpha\dot{\chi}^\beta = 0$ . That is to say, we are strictly interested in the inhomogeneous solution of the previous equation. Notice that the effective mass  $C^a_b$  contains a contribution from the Riemann tensor. However, the direction given by  $\dot{\phi}_*^a$  continues to be a flat direction since  $\mathcal{R}^a_{cdb}(\phi_*)\dot{\phi}_*^c\dot{\phi}_*^d\dot{\phi}_*^b = 0$ . In other words,

$$C^a_b(\phi_*)\dot{\phi}_*^b = 0. \quad (\text{B.20})$$

Additionally, notice that  $C_{ab}$  is symmetric. To proceed, let us define a few more quantities. First, the tangent vector to the trajectory defined by  $\phi_*(t)$  on the surface is given by

$$T_*^a = \frac{\dot{\phi}_*^a}{\dot{\phi}_*}, \quad (\text{B.21})$$

where  $\dot{\phi}_*^2 = \gamma_{ab}\dot{\phi}_*^a\dot{\phi}_*^b$ . In fact, notice that

$$T_*^a = X^a_\alpha T_*^\alpha, \quad (\text{B.22})$$

$$T_*^\alpha = \frac{\dot{\chi}^\alpha}{\dot{\phi}_*}, \quad (\text{B.23})$$

$$\dot{\phi}_*^2 = g_{\alpha\beta}\dot{\chi}^\alpha\dot{\chi}^\beta. \quad (\text{B.24})$$

It is a simple matter to show that

$$\frac{DT_*^a}{dt} = \dot{\phi}_* Q^a_{\alpha\beta} T_*^\alpha T_*^\beta, \quad (\text{B.25})$$

$$\dot{\phi}_* = 0. \quad (\text{B.26})$$

It follows that  $N_*^a \propto Q^a_{\alpha\beta} T_*^\alpha T_*^\beta$ . It should be clear that  $T_*^b V^a_b(\phi_*) = 0$ , as  $T_*^a$  is by definition along the flat directions of the potential. It is useful to consider the definition of the radius of curvature  $\kappa_*$  parametrising the deviation of the trajectory in  $S$  with respect to geodesics in  $\mathcal{M}$ . The radius of curvature  $\kappa_*$  comes defined as

$$\frac{DT_*^a}{d\phi_*} = -\frac{N_*^a}{\kappa_*}, \quad (\text{B.27})$$

and therefore one has

$$\frac{1}{\kappa_*} = -N_{*a} Q^a_{\alpha\beta} T_*^\alpha T_*^\beta = \sqrt{\gamma_{ab} Q^a_{\alpha\beta} T_*^\alpha T_*^\beta Q^b_{\gamma\delta} T_*^\gamma T_*^\delta}. \quad (\text{B.28})$$

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Notice that this quantity depends only on geometrical objects, as it should. Coming back to (eq. B.18), we may now write

$$\frac{D^2 \Delta^a}{dt^2} - \dot{\phi}_*^2 N_*^a \kappa_*^{-1} + C^a_b(\phi_*) \Delta^b = 0. \quad (\text{B.29})$$

At this point one may argue that there are no good reasons to consider  $\kappa_*^{-1}$  to be a small parameter. In fact, typically, for theories incorporating modular fields,  $\kappa$  should be of  $\mathcal{O}(1)$  in Planck units. Since  $\dot{\phi}_*$  is constant, it is convenient to parametrise the trajectory with  $\phi_*$ . We can in fact write

$$\frac{D \Delta^a}{dt} = \dot{\phi}_* \frac{D \Delta^a}{d\phi_*}, \quad (\text{B.30})$$

$$\frac{D^2 \Delta^a}{dt^2} = \dot{\phi}_*^2 \frac{D^2 \Delta^a}{d\phi_*^2}. \quad (\text{B.31})$$

We can therefore re-express the equation of motion for  $\Delta^a$  in terms of the proper parameter  $\phi_*$  along the curve:

$$\frac{D^2 \Delta^a}{d\phi_*^2} + \frac{1}{\dot{\phi}_*^2} C^a_b(\phi_*) \Delta^b = N_*^a \kappa_*^{-1}. \quad (\text{B.32})$$

To gain experience with this equation, consider the following situation. Suppose we have a trajectory in field space characterised by a constant curvature  $\kappa_*$  and such that  $C^a_b N^b = M^2 N^a$  with  $M^2 > 0$  a constant. That is,  $N^a$  is an eigenvector of  $C^a_b$ . Under such conditions, using the results of section 4.2.1 we find that

$$\frac{D^2 N^a}{d\phi_*^2} = -\frac{N^a}{\kappa_*^2}. \quad (\text{B.33})$$

Then, we can see that  $\Delta^a = \Delta N^a$  with  $\Delta$  constant is a solution of the equation, with

$$\Delta = \frac{\dot{\phi}_*^2}{\kappa_*} \left( M^2 - \frac{\dot{\phi}_*^2}{\kappa_*^2} \right)^{-1}. \quad (\text{B.34})$$

It is entirely reasonable to expect  $M^2 \gg \dot{\phi}_*^2 / \kappa_*^2$ , which corresponds to the case in which the energy scale of the low energy dynamics is much smaller than the energy scale associated to the heavy fields. In such a case we simply have

$$\Delta \simeq \frac{\dot{\phi}_*^2}{M^2 \kappa_*}, \quad (\text{B.35})$$

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This is the typical deviation from the true minimum of the potential if the surface of this minimum is not a geodesic, the deviation from which is parametrised by  $\kappa_*$ . To be more general, let us focus on a class of background trajectories in which

$$\frac{D\Delta^a}{d\phi_*} \sim O\left(\frac{\Delta}{\kappa_*}\right). \quad (\text{B.36})$$

This is a very reasonable situation to look into (our previous example is a particular case of this) as it correspond to those cases in which the main scale encoding the geometrical effects in the trajectory is its curvature. Then, if the non-vanishing eigenvalues of  $C^a_b$  are much larger than  $\dot{\phi}_*^2/\kappa^2$  we can neglect the first term in (eq. B.32) and write

$$C^a_b(\phi_*)\Delta^b \simeq \frac{\dot{\phi}_*^2 N^a}{\kappa_*}. \quad (\text{B.37})$$

Thus more generally  $\Delta \simeq \dot{\phi}_*^2/(M^2\kappa_*)$  is indeed a good measure of the deviation from the true minimum. Notice that in the case of a system with two scalar fields this is precisely the case.

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# Samenvatting

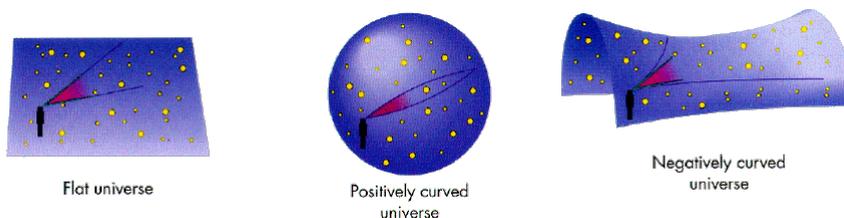
Als een systeem warmer wordt, worden meer vrijheidsgraden van een systeem toegankelijk. In ijs zijn bepaalde vibraties van het waterstofmolecuul niet beschikbaar, in vloeibaar water wel. Dit is goed zichtbaar door ijs en water in de magnetron te zetten. De magnetron zendt straling uit precies op de frequentie van zo'n vibratie en kan vloeibaar water daardoor efficiënt verwarmen. Ijs daarentegen is veel moeizamer te verwarmen in een magnetron.

In het heelal zijn er allerlei vrijheidsgraden die pas bij zeer hoge temperatuur waargenomen kunnen worden. Een paar daarvan kennen we, of denken we te kennen. Het Higgsdeeltje, waar de LHC in Genève naar zoekt, is een voorbeeld van een vrijheidsgraad die pas bij een hoge temperatuur toegankelijk is. Dat we denken te weten dat het Higgsdeeltje bestaat, komt omdat de aanwezigheid van deze vrijheidsgraad zich op lagere temperaturen verradert door correcties op bepaalde processen. Een aantal van zulke processen is nauwkeurig gemeten en de meting is alleen te verklaren als er een Higgsdeeltje is.

Echter, op hoge temperaturen zijn er waarschijnlijk vele vrijheidsgraden die we nog niet kennen. We weten nog niet welke theorie de natuur op de allerhoogste temperaturen verklaart. Wat we wel weten is dat alle tot dusver geponeerde kandidaat-theorieën gepaard gaan met vele nieuwe vrijheidsgraden. Om de theorie beter te leren begrijpen is het nodig deze nieuwe vrijheidsgraden te leren kennen. Deze vrijheidsgraden laten zich alleen zien onder bijzondere omstandigheden. Zulke omstandigheden kunnen we op aarde niet creëren, maar het heelal biedt mogelijkheden. Over wat we kunnen leren over deze vrijheidsgraden met behulp van het bestuderen van het heelal gaat dit proefschrift. Deze samenvatting beschrijft waarom het heelal zo'n goed meetinstrument is voor deze vrijheidsgraden, wat daarvan geleerd is en nog te leren valt.

## Ons heelal

Volgens de meest recente waarnemingen is ons heelal is  $13,7 \pm 0,4$  miljard jaar oud. Daarnaast is het heelal, op grote schaal bekeken, ruimtelijk erg vlak, isotroop en homogeen. Ruimtelijk vlak betekent dat evenwijdige lijnen altijd evenwijdig blijven. Op een gekromd oppervlak doen ze dat niet: bijvoorbeeld op een positief gekromd lichaam, zoals onze aarde, zullen evenwijdige lijnen elkaar altijd kruisen (figuur 1). Homogeen betekent dat het heelal overal ongeveer hetzelfde is. Dit geldt uiteraard niet voor kleine schalen. Een kubieke meter in het midden van een ster is duidelijk anders dan een kubieke meter vacuüm tussen de sterrenstelsels, maar de eigenschappen van een willekeurig gekozen kubus met een ribbe van 100 miljard lichtjaar<sup>1</sup> blijken opvallend gelijk aan de eigenschappen van elke andere willekeurig gekozen kubus. Isotroop betekent dat het heelal in alle richtingen gelijk is. Ook dit is uiteraard op kleine afstanden niet het geval, maar op dezelfde schaal van 100 miljard lichtjaar blijkt er geen richtingsvoorkeur meer te zijn.



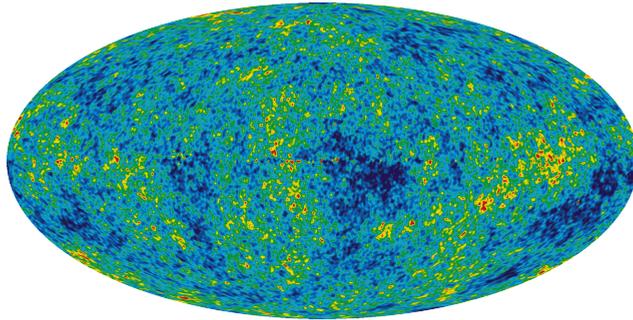
**Figuur 1:** *In een vlak heelal blijven evenwijdige lijnen evenwijdig. In gekromde ruimten is dit anders: in een positief gekromd heelal komen evenwijdige lijnen steeds dichterbij elkaar; in een negatief gekromd heelal wordt de afstand tussen twee evenwijdige lijnen juist steeds groter (bewerking van afbeelding van James Schombert).*

Dat ons heelal vlak, isotroop en homogeen is, is heel bijzonder. Dit laat zich goed illustreren met de eerste straling die vrij in het heelal kon reizen: de microgolfachtergrondstraling (figuur 2). Deze straling is ontstaan toen het heelal genoeg afkoelde om neutraal waterstofgas te vormen, ongeveer 380.000 jaar na de oerknal. In het vroege heelal was alle waterstof geïoniseerd, dat wil zeggen dat de positieve kern en de negatieve elektronen vrij van elkaar konden bewegen. Vrije elektronen zijn heel efficiënt in het verstrooien van licht. Alle lichtdeeltjes die in het heelal zaten werden constant verstrooid, en konden niet vrij reizen. Toen 380.000 jaar na de oerknal

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<sup>1</sup>De afstand die licht in één jaar aflegt is een lichtjaar. Dat is ongeveer 9,5 biljoen kilometer.

het heelal afkoelde tot beneden circa 3000 graden konden de elektronen zich aan de waterstofkernen binden. Waterstofatomen zijn veel minder efficiënt in het verstrooien van licht, zodat de lichtdeeltjes vanaf dat moment grotendeels onverstrooid naar onze telescopen konden reizen. Dit geeft ons een foto van de toestand van het heelal 380.000 jaar na de oerknal. Door de uitdijning van het heelal is deze straling met ongeveer een factor 1100 afgekoeld tot een temperatuur van 2,73 Kelvin (ongeveer  $-270,4^{\circ}\text{C}$ ) nu.



**Figuur 2:** De microgolf-achtergrondstraling zoals gemeten door de Wilkinson Microwave Anisotropy Probe (WMAP). De temperatuur van de straling is gemiddeld 2,73 K, ongeveer  $-270,4^{\circ}\text{C}$ . Bij de vorming van deze straling was de temperatuur nog ongeveer  $3000^{\circ}\text{C}$ , door de uitdijning van het heelal is de straling afgekoeld. De temperatuurverschillen die hier in een kleurcode zijn weergegeven zijn in de orde van een honderdduizendste graad.

Wat opvalt, zoals te zien in figuur 2, is dat de temperatuur van de straling overall ongeveer gelijk is. Dat is bepaald niet logisch: de straling die ons aan de noordpool bereikt heeft 13,7 miljard jaar lang naar ons gereisd. De straling die aan de zuidpool komt heeft een even lange reis gemaakt. De onderlinge afstand tussen de straling van de noord- en zuidpool is echter dubbel zo lang, 27,4 miljard lichtjaar. Deze punten kunnen elkaar op dit moment nog niet zien, en toch zijn ze even warm. Het is echter nog veel vreemder: we zien een plaatje van het heelal zoals het 380.000 jaar na de oerknal was, en licht kan op dat moment dan ook niet meer dan 380.000 lichtjaar afgelegd hebben. We zouden verwachten dat er geen gebieden zijn groter dan 380.000 lichtjaar, de afstand die licht gereisd kan hebben tussen de oerknal en de vorming van de microgolf-achtergrondstraling, die dezelfde temperatuur hebben. Een gebied van 380.000 lichtjaar ten tijde van de vorming van de microgolf-achtergrondstraling blijkt ongeveer één graad aan de huidige hemel te beslaan. Dat betekent aan op de

huidige hemel 40.000 gebieden zijn met een oppervlak van een vierkante graad zijn die precies dezelfde temperatuur hebben, ondanks dat ze niet met elkaar in contact stonden toen ze hun temperatuur-signaal uitzonden.

Als we beter naar het signaal kijken, blijken er gecorreleerde rimpels te zijn, zoals te zien in figuur 1.3 in de introductie. De eerste piek in dit figuur is gerelateerd aan de afstand die geluid sinds de oerknal kon reizen in het heelal, een geluidsgolf ter grootte van de geluidshorizon was precies op haar maximum. De andere pieken worden gevormd door boventonen. Echter, ook op afstanden groter dan de geluidshorizon, tot de grootte van ons zichtbare heelal, blijkt uit figuur 1.3 dat het signaal gecorreleerd is. Dit lijkt in strijd met het idee dat geen signaal meer afstand kan hebben afgelegd dan de afstand die licht kan hebben afgelegd sinds de oerknal.

Ook een vlak heelal is ongewoon. Zwaartekracht heeft de neiging materie te klonteren en zo de kromming te laten toenemen. Dat betekent dat het heelal, om nu ongeveer vlak te zijn, vroeger veel vlakker geweest moet zijn. Een vlak heelal betekent dat de bewegingsenergie van de materie precies even groot is als de zwaartekrachtsenergie die deze materie bij elkaar houdt. Om het heelal nu zo vlak te krijgen als is waargenomen, moet de afstemming tussen bewegings- en zwaartekrachtsenergie vroeger extreem precies geweest zijn. Zo moet ten tijde van de vorming van de achtergrondstraling het verschil kleiner dan vier miljoenste procent zijn geweest,  $10^{-35}$  seconde na de oerknal (in de volgende paragraaf wordt duidelijk wat er op dit tijdstip gebeurde) was dit verschil zelfs kleiner dan 1 op  $10^{-27}$ . Ter illustratie: als met deze precisie een raket naar de dichtstbijzijnde ster<sup>2</sup> wordt gestuurd zou de raket bij die ster op de schaal van ongeveer één atoom, een tien-miljardste meter, nauwkeurig aankomen.

Kort samengevat is het heelal erg gevoelig voor de beginparameters. Om het heelal te krijgen zoals het nu is vergt een zeer precieze instelling van deze parameters. Dit heet finetuning, veel finetuning wordt over het algemeen gezien als een probleem. Ons heelal heeft veel finetuning nodig, tenzij een fysisch mechanisme gevonden wordt om deze finetuning automatisch te laten plaatsvinden. Dit mechanisme is gevonden: inflatie.

## Inflatie

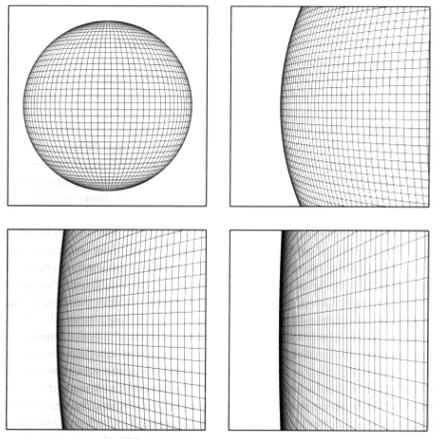
In 1980 is door Alan Guth het idee geopperd dat het heelal vlak na de oerknal exponentieel snel is uitgedijd, inflatie. Deze zeer snelle uitdijning blijkt de finetuning-

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<sup>2</sup>De dichtstbijzijnde ster is natuurlijk de zon. Ik bedoel hier echter Proxima Centauri, op een afstand van 4,2 lichtjaar.

problemen zoals in de vorige paragraaf beschreven te kunnen oplossen. Inflatie wordt veroorzaakt doordat de vacuümenergie van het heelal positief is. Als je Einstein's vergelijkingen oplost met een vacuümenergieterm, blijkt dat te leiden tot een exponentieel versnelde uitdijning.<sup>3</sup> En dat heeft vele gevolgen.

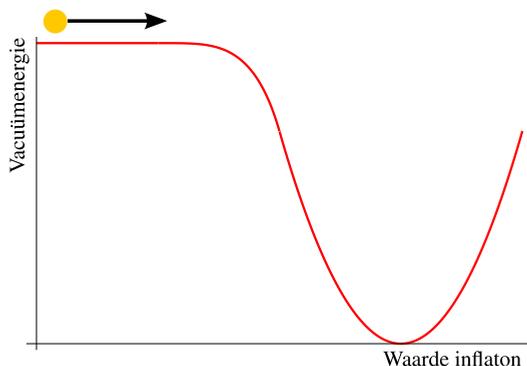
Een eerste gevolg van inflatie is dat de afstand tussen twee punten sneller groeit dan de afstand die licht kan reizen. Dit is geïllustreerd in figuur 1.2(a) in de introductie. Dit maakt het mogelijk dat het hele huidige zichtbare heelal vóór inflatie een zo klein gebied besloeg, dat licht wel de tijd had om van de ene naar de andere kant in dit gebiedje te reizen. Zo kon vóór inflatie dit gebiedje in thermisch evenwicht raken en kunnen we verklaren waarom de temperatuur van de microgolf-achtergrondstraling overal ongeveer hetzelfde is. Immers, de hele zichtbare achtergrond is ooit in thermisch contact geweest.



**Figuur 3:** In een exponentieel versnelde uitdijend heelal wordt de kromming steeds kleiner (afbeelding van Margaret Hanson).

Een tweede gevolg van inflatie is dat het heelal vlakker wordt, dat de bewegings- en zwaartekrachtsenergie steeds gelijkjer worden. De zwaartekrachtsvergelijking heeft

<sup>3</sup>Einstein gebruikte deze waarneming om een kosmologische constante toe te voegen, waarbij deze versnellingsterm precies de door de zwaartekracht van de materie veroorzaakte vertraging moest opheffen, zodat een stabiel heelal mogelijk werd. Later werd dit idee door hem zijn "grootste blunder uit mijn carrière" genoemd, nadat Willem de Sitter had aangetoond dat met dit idee geen stabiel heelal beschreven kan worden. Enige jaren later, in 1929, toonden waarnemingen van Edward Hubble aan dat het heelal niet statisch is, maar uitdijt. Echter, hoewel deze door Einstein voorgestelde term niet voldoet om een statisch heelal te maken, voldoet hij wel om een exponentieel snel uitdijend heelal te beschrijven.



**Figuur 4:** *Inflatie wordt veroorzaakt door een veld dat gedurende een bepaalde tijd een grote vacuümenergie genereert. Inflatie wordt beëindigd doordat het veld ophoudt deze vacuümenergie te genereren, en deze energie uiteindelijk terechtkomt in de materie waaruit het huidige heelal bestaat.*

normaal als oplossing dat de kromming van een gekromd object groter wordt, in een exponentieel uitdijend heelal draait dit om en wordt de kromming juist steeds kleiner (figuur 3). Zo zal genoeg inflatie een erg vlak heelal opleveren, of de bewegings- en zwaartekrachtsenergie erg gelijk maken, gelijkjer dan de vereiste maximale afwijking van 1 op  $10^{-27}$ .

Een derde gevolg van inflatie, het gevolg waarvan in dit proefschrift veelvuldig gebruik wordt gemaakt, is dat inflatie ook de rimpels voorspelt. Deze rimpels zullen na inflatie als geluidsgolven door het heelal reizen en de waargenomen gecorreleerde rimpels in de microgolf-achtergrondstraling maken (figuur 2 en 1.3). Om het ontstaan van de rimpels te begrijpen is het nodig in meer detail naar het mechanisme dat inflatie veroorzaakt te kijken.

Zoals beschreven leidt een vacuümenergie tot een exponentieel versnelde uitdijing van het heelal. Wat we nodig hebben is een mechanisme om zo'n vacuümenergie voor een bepaalde tijd te genereren. Als dit mechanisme eindigt zal de vacuümenergie vervallen in de materie waar wij van gemaakt zijn, en zal inflatie eindigen. Om dit te bereiken is een veld nodig dat een vacuümenergie genereert als het niet in de toestand van minste energie zit. Verder moet er wel langzaam een ontwikkeling naar de toestand van minimale energie plaatsvinden. Dit kan door een veld te kiezen dat langdurig niet in de toestand van minimale energie zit, zoals in figuur 4 is weergegeven. Het veld bevindt zich tijdens inflatie op het vlakke stuk met grote vacuümenergie,

om aan het einde van inflatie naar het nulpunt te vallen.

Het veld dat nodig is om inflatie te genereren heet het inflaton. Dit veld ontwikkelt zich langzaam en daarbij zal het zich volgens de wetten van de quantummechanica gedragen. Een van de belangrijkste eigenschappen van de quantummechanica is dat alles een beetje onzeker is. Dat betekent dat ook de waarde van het veld niet helemaal zeker is en dus van plaats tot plaats een klein beetje anders zal zijn. Dit zorgt voor quantumrimpels, die vervolgens door de snelle uitdijning van het heelal kunnen worden opgeblazen tot de grootte van ons zichtbare heelal en daarmee de rimpels in de microgolf-achtergrondstraling veroorzaken.

Om dit hele mechanisme te laten werken is het nodig dat het heelal in hele, hele korte tijd heel, heel veel groter wordt. Om precies te zijn, in zo'n  $10^{-35}$  seconde moet het heelal minimaal  $10^{24}$  keer zo groot worden. Als dit proefschrift met een vergelijkbare factor vergroot zou worden, zou het boekje de grootte van het zichtbare heelal krijgen. Zo'n extreme uitdijning stelt eisen aan het veld dat inflatie veroorzaakt. De vacuümenergie moet, ondanks dat het heelal zo snel uitdijt, erg constant blijven, zodat de gevormde rimpels overeenkomen met de waarnemingen van deze rimpels in de microgolf-achtergrondstraling. Deze eis wordt "slow-roll"-inflatie genoemd. Het blijkt verrassend eenvoudig om een vergelijking voor één enkel veld op te schrijven zodanig dat het voldoet aan de eis van "slow-roll" en rimpels voorspelt die overeenkomen met de waarnemingen. Echter, in ons heelal zit veel meer dan één veld.

## Hoge-energiefysica van inflatie

De fysica van de deeltjes waarvan wij gemaakt zijn, elektronen en quarks,<sup>4</sup> evenals de krachten die deze deeltjes beïnvloeden<sup>5</sup> wordt beschreven met deeltjes en velden. Op lage temperatuur zijn er dan ook al een heleboel velden. Als de temperatuur opgevoerd wordt, wordt de situatie niet beter. Dit omdat bij hogere temperatuur juist meer in plaats van minder vrijheidsgraden beschikbaar komen.

Op dit moment is er nog geen theorie om zowel de fysica op de kleinste schalen, het niveau van elektronen en quarks, als op de grootste schalen, het niveau van sterrenstelsels en het heelal, te beschrijven. Op de kleinste schalen zijn de quantumtheorieën voor elektromagnetisme, de zwakke kracht en de sterke kracht erg goed beschreven door het standaardmodel. Op de grootste schalen is alleen zwaartekracht

<sup>4</sup>Quarks zijn de deeltjes waarvan protonen en neutronen gemaakt zijn. Protonen en neutronen vormen de bouwstenen van atoomkernen, welke samen met elektronen de atomen vormen.

<sup>5</sup>Elektromagnetisme, de sterke kracht die ervoor zorgt dat atoomkernen bijeen blijven en de zwakke kracht welke zorgt voor radioactiviteit.

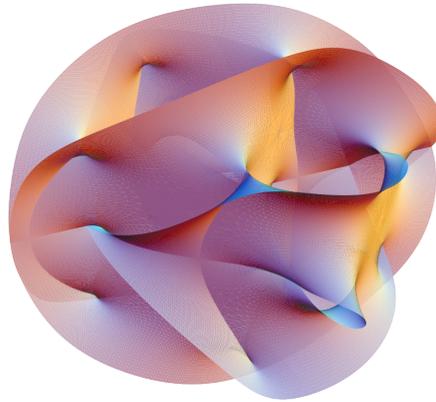
relevant, welke nauwkeurig beschreven wordt door Einstein's algemene relativiteitstheorie, waarvan geen quantumbeschrijving bestaat. Doordat nog niet bekend is hoe zwaartekracht met quantummechanica gecombineerd moet worden is er nog geen beschrijving van de zwaartekracht op de schaal van het standaardmodel.

Wel zijn er een aantal voorstellen voor een theorie die tegelijkertijd het standaardmodel en de algemene relativiteitstheorie moet beschrijven. De belangrijkste kandidaat is de snaartheorie. In deze theorie wordt als fundamentele bouwsteen geen puntdeeltje, maar een heel klein snaartje gebruikt. Dit maakt het mogelijk om de problemen met zwaartekracht op de kleinste schalen te omzeilen. Snaartheorie blijkt in drie dimensies niet goed te werken. De problemen met snaartheorie zijn echter op te lossen in een ruimte met negen dimensies.<sup>6</sup> Dit is uiteraard in tegenstelling met de waarnemingen van onze overduidelijk driedimensionale wereld. Onze waarnemingen zijn echter beperkt tot schalen die we daadwerkelijk kunnen zien. We weten dat het heelal drie grote ruimtelijke dimensies heeft, maar het is best mogelijk dat er ook nog een aantal kleine ruimtelijke dimensies zijn. Vergelijk dit met een lang touw: van dichtbij is een touw duidelijk driedimensionaal, met een lengte langs het touw en een dikte in de tweedimensionale doorsnede. Als het touw van veraf bekeken wordt, is de dikte echter niet meer te zien. Ook voor de fysica van het touw kunnen de "kleine dimensies" onbelangrijk worden. Als je golven in het touw maakt, maakt de dikte van het touw niet zoveel uit. Samen met het soort touw bepaalt de dikte de stijfheid, maar verder is voor het beschrijven van de golf alleen een coördinaat in de lengte van het touw nodig. Pas bij korte golven wordt het mogelijk dat het touw ook in de breedte gaat bewegen waardoor de "kleine dimensies" zichtbaar worden.

Om van de negen ruimtelijke dimensies van snaartheorie naar de drie ruimtelijke dimensies van onze wereld te komen, is een methode nodig om zes dimensies heel klein te maken, "compactificatie". Om dit voor elkaar te krijgen wordt aangenomen dat de negendimensionale ruimte bestaat uit een driedimensionale ruimte met op elk punt een piepkleine zesdimensionale ruimte (figuur 5). De groottes en vormen van deze zesdimensionale ruimte liggen niet vast, maar kunnen variëren. Dit levert extra vrijheidsgraden op, die in onze driedimensionale wereld zichtbaar zijn als zogeheten scalaire velden. Deze scalaire velden zijn tot dusver nog niet gevonden, wat betekent dat de groottes en vormen van deze zesdimensionale ruimte erg stabiel moeten zijn. Dit is niet de natuurlijke toestand, een probleem waarvoor pas in het begin van deze eeuw een eerste deeloplossing is gepresenteerd. Er is ontdekt dat deze zesdimensionale ruimte klein gehouden kan worden door equivalenten van elektromagnetische velden en ladingen toe te voegen, omdat het energie kost om van een ruimte met een

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<sup>6</sup>Meestal wordt de snaartheorie tiendimensionaal genoemd. Dit zijn de negen ruimtelijke dimensies plus tijd.



**Figuur 5:** Een projectie van een zesdimensionale ruimte van het soort dat gebruikt wordt voor compactificatie. De vele vormen en volumes van zowel de gehele ruimte alsmede de “gaten” vormen vrijheidsgraden (afbeelding van Andrew Hanson).

elektromagnetisch veld de vorm of het volume te veranderen.

Een van de bekendste mechanismen om dit voor elkaar te krijgen is het mechanisme van Kachru, Kallosh, Linde en Trivedi (KKLT) uit 2003. Bezwaar op het KKLT-mechanisme is dat de oplossing in meerdere stappen uitgevoerd wordt, waarbij bij iedere stap ervan uitgegaan wordt dat de volgende stap onafhankelijk is van eerdere stappen. In hoofdstuk 2 hebben wij laten zien dat dit over het algemeen niet het geval is en dat grote correcties op het KKLT-mechanisme te verwachten zijn. Een ander mechanisme om vorm en volume van de zesdimensionale ruimte vast te leggen is het zogeheten grote-volume scenario.<sup>7</sup> Dit scenario gaat uit van eenzelfde stappenplan als het KKLT-mechanisme om uiteindelijk alle vormen en het volume stabiel te maken. Echter, in dit model blijken de correctietermen veel kleiner te zijn, doordat deze correctietermen kleiner worden als het volume groter wordt.

In hoofdstuk 3 bekijken we een model in de superzwaartekracht-theorie en bestuderen we hoe in superzwaartekracht een vacuümenergieterm kan worden toegevoegd. Superzwaartekracht kan gebruikt worden om op lage energieën snaartheorie te beschrijven. Een van de problemen van snaartheorie en superzwaartekracht is dat ze een

<sup>7</sup>Groot volume betekent hier groot ten opzichte van de grootte van de snaren, die ongeveer  $10^{-33}$  meter groot zijn. Voor onze begrippen is dit grote volume echter nog heel erg klein, van de orde van  $10^{-23}$  meter, te klein om met de LHC in Genève te detecteren.

ruimte beschrijven met een negatieve vacuümenergie. Voor inflatie hebben we echter een positieve energie nodig, in het huidige heelal moet de vacuümenergie vrijwel nul zijn.<sup>8</sup> Om dit voor elkaar te krijgen is een “uplifting”-term nodig. Een probleem van een “uplifting”-term is echter dat heel vaak de theorie instabiel wordt als een “uplifting”-term wordt toegevoegd. Wij hebben laten zien dat het mechanisme dat stabiele theorieën instabiel maakt een bepaalde klasse van instabiele theorieën juist stabiel kan maken na “uplifting”.

In de laatste twee hoofdstukken, hoofdstukken 4 en 5, kijken we naar mogelijkheden om de eigenschappen van de extra kleine dimensies te achterhalen. We doen dit in de specifieke context van inflatie, omdat deze correctietermen hier de grootste rol spelen. Intuïtief is het logisch: als drie dimensies  $10^{24}$  keer zo groot worden, waarom doen de overige zes dimensies dan niet mee? Zoals beargumenteerd in hoofdstuk 2 is het niet eenvoudig de kleine dimensies klein te houden. Mechanismen die deze dimensies wel klein houden hebben meestal correctietermen nodig. Deze correctietermen kunnen een grote rol spelen tijdens inflatie. Dat kan doordat deze termen correcties geven op de waarde van het inflatonveld dat inflatie veroorzaakt, zodat feitelijk inflatie niet beschreven wordt door één veld maar door meerdere. In het geval meerdere van deze velden ongeveer even zwaar zijn en daardoor makkelijk kunnen bijdragen aan inflatie, is sprake van meervelden-inflatie. Deze vorm van inflatie wordt door veel mensen uitgebreid bestudeerd en levert interessante manieren op om de theorie te testen met behulp van waarnemingen van ons heelal.

Wij hebben laten zien dat zelfs als de velden die het inflatonveld beïnvloeden veel zwaarder zijn, ze ook effecten opleveren. De algemene aanname is dat de bijdrage van zulke zware toestanden sterk onderdrukt is. Echter, in het geval van inflatie kunnen deze toestanden in belangrijke mate het traject bepalen waarlangs de waarde van het inflatonveld zich ontwikkelt. Dit kan ervoor zorgen dat dit traject bochten maakt (omslag en figuur 4.1). Zulke bochten blijken in potentie waarneembare signalen in de microgolf-achtergrondstraling achter te laten. Dit biedt de mogelijkheid om met de studie van de microgolf-achtergrondstraling ook de fysica van zulke zware toestanden te achterhalen. Dit is een van de belangrijkste resultaten van dit proefschrift.

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<sup>8</sup>Recente waarnemingen laten zien dat ook in het huidige heelal de vacuümenergie positief is, omdat ons heelal inmiddels weer versneld uitdijt. Deze energieterm is echter wel veel en veel kleiner dan de vacuümenergie tijdens inflatie.

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# Summary

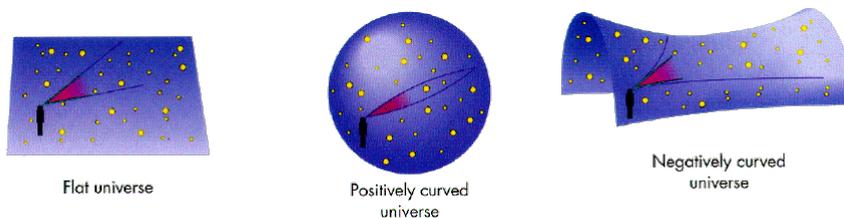
If a system heats up, more degrees of freedom become available. In ice certain vibrations of the water molecule are not available, yet in liquid water they are. This can be made visible by putting both ice and water in a microwave oven. The microwave emits radiation exactly at the wavelength of one of these vibrations and is therefore very efficient in heating water. On the other hand, heating ice with a microwave oven takes much longer.

In our universe there are many degrees of freedom that can be made visible only at very high temperatures. Some of those we know, or at least we think we know. The Higgs particle, the particle the LHC in Geneva is looking for, is a well known example of a degree of freedom that is observable only at very high temperatures. We think we know the Higgs particle exists because it betrays its existence at lower temperatures due to certain corrections to specific processes. Some of these processes have been measured very accurately and the measurements can be explained only by assuming the existence of a Higgs particle.

There are likely many more degrees of freedom at very high temperatures that we do not yet know. We currently do not know which theory explains the physics at the highest temperatures. What we do know is that all proposed candidates predict many new degrees of freedom. In order to understand such theories, it is important to know the degrees of freedom associated to it. These degrees of freedom are visible only under very special circumstances. Such circumstances are not available on earth, but our universe offers possibilities. This thesis is about what we can learn from these degrees of freedom by studying the properties of our universe. Why it is that our universe is such an exquisite measuring instrument for these degrees of freedom, what we have learned and can still learn about them is the subject of this summary.

## Our universe

According to the latest measurements, our universe is  $13.7 \pm 0.4$  billion ( $10^9$ ) years old. Furthermore, our universe is at large scales spatially flat, homogeneous and isotropic. Spatially flat means that parallel lines will stay parallel forever. On a curved surface parallel lines will not stay parallel. For example, on a positively curved surface, such as the surface of the earth, parallel lines shall always cross (figure 6). Homogeneous means that the universe is the same everywhere. This is of course not meant to be true at small scales: a cubic metre in the middle of a star is clearly different from a cubic metre in the emptiness of outer space. Yet, if we compare the properties of a randomly chosen cube with sides of approximately 100 million light years<sup>9</sup> such cubes turn out to be very similar to any other randomly chosen cube. Isotropy means that the universe is the same in all directions, which of course must be understood also to be valid only at scales of about 100 million light years.



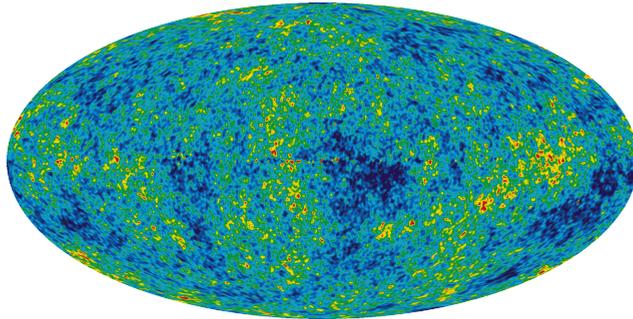
**Figure 6:** *In a flat universe parallel lines stay parallel. In a curved space this is different: in a positively curved space parallel lines will close up on each other, in a negatively curved universe the distance between parallel lines will keep increasing (edited version of an image from James Schombert).*

A flat, homogeneous and isotropic universe is very special. An apt illustration is provided by the first radiation that could travel freely through our universe, the microwave background radiation (figure 7). This radiation is emitted when the universe cooled down below the point where hydrogen was ionised, which happened approximately 380,000 years after the big bang. In the early universe all hydrogen was ionised, its positive nucleus and negative electron were not bound together but formed a charged plasma in which they could move freely. The free electrons in that plasma are very efficient in scattering light. Therefore, all particles of light were constantly scattered and thus could not travel freely. Then, 380,000 years after the big

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<sup>9</sup>A light year is the distance light travels in one year, which is approximately 9.5 trillion ( $10^{12}$ ) kilometres.

bang, the universe cooled down enough, to about  $3000^{\circ}\text{C}$ , to allow the electrons to bind to the hydrogen nuclei and form atomic hydrogen. Atomic hydrogen is much less efficient in scattering light, thereby allowing the particles of light to travel mostly unscattered to our telescopes. This radiation thus provides us with a picture of the universe as it was 380,000 years after the big bang. Due to the expansion of the universe this radiation has cooled down by about a factor 1100, to a temperature of 2.73 Kelvin ( $-270.4^{\circ}\text{C}$ ) today.



**Figure 7:** *The microwave background radiation as observed by the Wilkinson Microwave Anisotropy Probe (WMAP). The temperature of the radiation is on average 2.73 K, about  $-270.4^{\circ}\text{C}$ . When emitted, this radiation had a temperature of about  $3000^{\circ}\text{C}$  but has cooled down to the current temperature due to the expansion of the universe. The temperature difference is displayed in false colors and represents temperature differences in the order of a hundred thousandth of a degree.*

From figure 7 it is evident that the temperature of this radiation is approximately equal everywhere. This is very nonsensical: the radiation that is received at say the north pole took 13.7 billion years to travel to us, the same holds for the radiation that is received at the south pole. Therefore, the distance between the points observed at both poles is 27.4 billion light years. These points can currently not see each other, and yet the temperatures are equal. Looking at it more closely, it is actually a lot stranger. As this radiation was emitted 380,000 years after the big bang at that time light could not have travelled more than 380,000 light years. Therefore, we would not expect areas larger than 380,000 light years at the time of the emission of the microwave background radiation to share the same temperature. Such an area turns out to cover about one degree of our current sky. This means there are about 40,000 independent areas with a surface area of one square degree that all have the same temperature, despite the fact that they have never been in causal contact at the time

## Summary

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they emitted their temperature signal.

When we look closer at this signal, it turns out that there are correlated ripples, as can be seen in figure 1.3 in the introduction. The first peak is related to the distance sound has travelled since the big bang, a sound wave at the size of the horizon would have been at its maximum. The other peaks are the higher harmonics. Yet, at distances larger than the sound horizon, up to the size of our current visible universe, the signal is found to be correlated. This appears to violate the idea that, since the big bang, no signal can have covered more distance than light.

Also a flat universe is uncommon. Gravity has the tendency to amplify existing concentrations of matter, which increases the curvature. That means that a universe that is flat now must have been very flat in the past. A flat universe means that the kinetic energy of matter is equal to the gravitational energy that keeps that matter together. In order to have a universe that is as flat as observed today the balance between kinetic- and gravitational energy must have been very finely tuned. For example, when the background was created, the allowed imbalance is of the order of four millionth of a percent.  $10^{-35}$  seconds after the big bang (in the next paragraph the meaning of this number will become clear) the energies needed to be balanced by less than one part in  $10^{27}$ . To illustrate, if a rocket is sent with a similar precision to the most nearby star,<sup>10</sup> the rocket will still be aimed with the precision of the radius of an atom, one ten-billionth of a metre.

Summarising, we can say that our universe is very sensitive to the initial parameters. To get a universe that is as our universe today requires a very precise tuning of these parameters, which is called finetuning. When much finetuning is needed, it is usually considered to be unnatural and problematic. Our universe requires a lot of finetuning, unless a physical mechanism can be found that automatically tunes the parameters to the required values. This mechanism has been found: inflation.

## Inflation

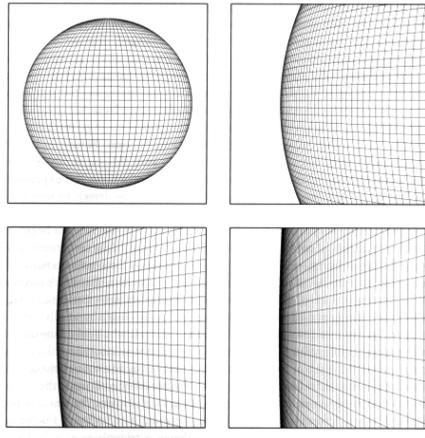
In 1980 Alan Guth proposed the idea that just after the big bang our universe went through a period of exponential expansion, inflation. This very rapid expansion can solve the issues with finetuning, as described in the previous paragraph. Inflation is caused by a positive vacuum energy in our universe. Solving Einstein's equations in the presence of a positive vacuum energy, one finds as a solution an exponential

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<sup>10</sup>The most nearby star is of course our sun. Yet here I am referring to Proxima Centauri, at a distance of 4.2 light years.

accelerated expansion.<sup>11</sup> This has many consequences.

A first consequence of inflation is that the distance between two points can grow faster than the distance light can travel. This is illustrated in figure 1.2(a) in the introduction. This allows the possibility that the current observable universe before inflation covered such a small area that light at that point in time was able to cross the entire visible universe. This allowed the universe to attain thermal equilibrium before inflation, explaining why the microwave background radiation has the same temperature everywhere as the whole observable universe has been in thermal contact.



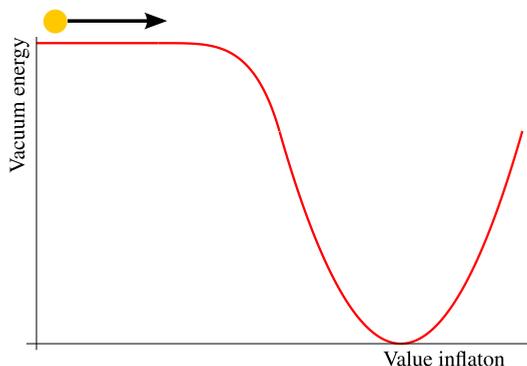
**Figure 8:** *In an exponential accelerated expanding universe the curvature decreases (image from Margaret Hanson).*

A second consequence of inflation is that the universe becomes increasingly flat, that kinetic and gravitational energy become increasingly balanced. The equations of general relativity normally yield a curvature that increases with time, in an exponentially expanding universe the curvature instead decreases with time (figure 8). Given enough inflation, the universe will be very flat and the kinetic and gravitational energy will be very balanced, up to the required one part in  $10^{27}$ .

<sup>11</sup>Einstein had used this observation to add a cosmological constant to his equations, such that the acceleration term exactly cancels the deceleration caused by gravity. This would allow a static universe. He later called this idea his “biggest blunder of my career”, as it was found by Willem de Sitter to be incapable of explaining a static universe. Several years later, in 1929, Edward Hubble discovered that the universe is not static but expanding. Yet, while Einstein’s proposed term does not allow a static solution of our universe, it does allow an exponential accelerated expanding one.

## Summary

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**Figure 9:** *Inflation is caused by a field that during some period in time generates a large vacuum energy. Inflation ends when this field stops generating this vacuum energy and eventually dumps all its energy into the matter that currently fills the universe.*

A third consequence of inflation, the consequence this thesis is partially built upon, is that inflation predicts ripples. After inflation, these ripples will travel as sound waves through the universe to generate the observed correlated ripples in the microwave background radiation (figure 7 and 1.3). In order to understand the generation of these ripples, it is first needed to study the mechanism that drives inflation.

As stated, a vacuum energy leads to an exponentially accelerated expansion of the universe. We now turn to the mechanism that can generate such a vacuum energy for a period of time. When this mechanism stops working inflation will end and the vacuum energy will turn into the particles that we are made of. In order to achieve this, we need a field that generates a vacuum energy when it is not in its state of minimal energy. Furthermore, a slow development towards the state of minimum energy is required. This can be achieved by a field that can spend long periods of time outside its state of minimum energy, as is depicted in figure 9. During inflation, the field will sit on the flat elevated stretch, in order to fall down the abyss at the end of inflation.

The field required to generate inflation is called the inflaton. This field develops slowly and has to obey the laws of quantum mechanics. One of the most important properties of quantum mechanics is that everything is somewhat uncertain. This means that the value of the field will also be somewhat uncertain and can thus differ from place to place. This generates quantum ripples, that due to the rapid expansion of the universe will be blown up to the size of our visible universe and thereby form

the ripples in the microwave background radiation.

For this mechanism to work it is needed that the universe in a very, very short time expands by a very, very huge amount. In numbers, in about  $10^{-34}$  seconds the universe must expand by a factor  $10^{24}$ . Expanding this thesis by a similar amount, it would end up filling the current visible universe. Such an extremely rapid expansion places strong requirements on the field that drives inflation. The vacuum energy must, despite the rapid expansion of the universe, remain very constant in order to create ripples that match the observed ripples in the microwave background radiation. The needed requirement is called the “slow-roll” condition. It turns out to be very simple to write down a single field equation that satisfies this condition. The problem is that our universe contains more than one field.

## High energy physics and inflation

The physics of the particles that we are made from, electrons and quarks,<sup>12</sup> as well as the forces that influence these particles<sup>13</sup> is described by particles and fields. At low temperatures there are already many fields. Increasing the temperature, the situation all but improves, as at higher energy more instead of fewer degrees of freedom become available.

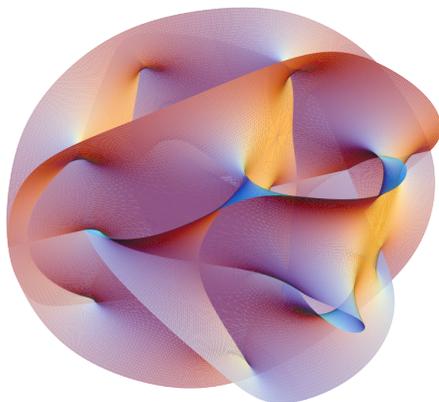
At this moment we do not know a theory that can simultaneously describe the physics at the smallest scales, the scales of electrons and quarks, and the largest scales, the scale of galaxies and our universe. At the smallest scales the quantum theories for electrodynamics, the weak and the strong force are very accurately described by the standard model. At the largest scales the only relevant force is gravity, which is accurately described by Einstein’s theory of general relativity, but does not have a quantum version. Because it is currently not known how to combine general relativity with quantum mechanics there is to date no description of gravity at the scales of the standard model.

Yet, we do have candidates for a quantum theory that should describe both the standard model and general relativity. The main candidate is string theory. In this theory, the fundamental building block is not a point particle but a tiny string. This allows to overcome the issues with gravity at the smallest scales. However, it turns out that string theory does not work properly in three dimensions. The problems can be

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<sup>12</sup>Quarks are the particles that protons and neutrons are made of. In turn, protons and neutrons are the building blocks of atomic nuclei that together with electrons make atoms.

<sup>13</sup>Electromagnetism, the strong force that keeps atomic nuclei together and the weak force that causes radioactivity.



**Figure 10:** *A projection of a six dimensional space of the kind that is used for compactification. The many sizes and shapes of both the overall volume and the “holes” are degrees of freedom (image from Andrew Hanson).*

solved in a space with nine dimensions.<sup>14</sup> This of course contradicts our observations of our obviously three dimensional world. Yet, our observations are limited to the scales we can actually measure. We know the universe has three large dimensions, but it might just be that there are also a number of small dimensions. Compare this to a long rope: at close the rope clearly is three dimensional, with a length along the rope and a thickness along the two dimensional cross section. Yet, if looked upon from afar, the thickness is no longer visible. Also for the physics of the rope the “small dimensions” can become irrelevant. If you make a wave in the rope the thickness usually does not matter much. Together with the material the thickness will determine the stiffness, but for describing the wave the only relevant degree of freedom is a coordinate along the length of the rope. Only at very short wavelengths it will be possible to have thickness distortions which will make the “small dimensions” visible.

In order to go from the nine spatial dimensions of string theory to the three spatial dimensions of our world, we need a method to make six dimensions very small, “compactification”. To achieve this, it is assumed that the nine dimensional space is made of our three dimensional space with at each point a very small six dimensional

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<sup>14</sup>Often, string theory is said to live in a ten dimensional space. These are nine spatial dimensions and time.

space (figure 10). The size and shape of this six dimensional space are not fixed, but can vary from point to point. This gives additional degrees of freedom that in our three dimensional world will be visible as so-called scalar fields. So far, these fields have not been found, which means that the sizes and shapes of this six dimensional space must be very stable. This is not the natural state, a problem that has only been partially solved at the beginning of this century. It has been discovered that this six dimensional space can be kept small by adding equivalents of electromagnetic fields and charges, because changing the shape or size of a space with an electromagnetic field requires energy.

One of the best known mechanisms to achieve this is the mechanism proposed in 2003 by Kachru, Kallosh, Linde and Trivedi (KKLT). An issue with this mechanism is that it uses a stepped approach to reach the solution, where each step is assumed to be independent from the previous steps. In chapter 2 we have shown that this is in general not the case and that large corrections to this mechanism can be expected. Another mechanism to fix the sizes and shapes of the six dimensional space is the so-called large volume scenario.<sup>15</sup> This scenario uses a similar stepwise approach as the KKLT mechanism to stabilize the sizes and shapes. Yet, in this model the corrections turn out to be much smaller because these corrections are getting smaller when the volume gets larger.

In chapter 3 we look at a model in the supergravity-theory and study how in supergravity a vacuum energy can be added. Supergravity can be used as a low energy description of string theory. A problem of both string theory and supergravity is that they describe spaces with a negative vacuum energy. Inflation requires a positive vacuum energy, in our current universe the vacuum energy is almost identically zero.<sup>16</sup> In order to achieve this, an “uplifting”-term has to be added. The problem with such an “uplifting”-term is that often the theory will become unstable after adding such a term. We have shown that the mechanism that turns stable theories unstable also allows the opposite, and turn a specific class of unstable theories stable after “uplifting”.

In the last two chapters, chapters 4 and 5, we look at the possible means to find out the nature of the extra small dimensions. We do this in the specific context of inflation, because during inflation the correction terms play the most important role.

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<sup>15</sup>Large volume here means large with respect to the size of individual strings which are about  $10^{-33}$  meter in size. From our point of view this large volume is still very small, in the order of  $10^{-23}$  meter, which is too small to be detectable by the LHC in Geneva.

<sup>16</sup>Recent observations have shown that also in our current universe there is a positive vacuum energy, because the expansion of our universe is currently accelerating. However, the required energy term is much, much smaller than the vacuum energy during inflation.

## Summary

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Intuitively, this makes sense: if three dimensions get  $10^{24}$  times larger, why do the other six dimensions not participate? As argued in chapter 2 it is not easy to keep the small dimensions small. Mechanisms that do keep these dimensions small generally require extra corrections. Such corrections can play a big role during inflation, to the extent that inflation is not described by only one field but by many. If these fields are approximately equally massive and thus can easily contribute to inflation we are in the domain of multi-field inflation. This kind of inflation is studied by many people and results in interesting means to test the theory with observations of our universe.

We have shown that even when the fields that influence the inflaton field are much more massive they also have consequences. The common assumption is that contributions from such very massive degrees of freedom are strongly suppressed. Yet, during inflation these degrees of freedom can determine up to a very large degree what is the trajectory along which the inflaton field develops. This can cause the inflaton field to experience turns (cover image and figure 4.1). Such turns turn out to generate potentially observable signals in the microwave background radiation. This offers the opportunity to study the physics of these very massive degrees of freedom by studying the microwave background radiation. This is one of the main conclusions of this thesis.

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# Dankwoord

Hier is het dan, het dankwoord. Vaak als belangrijkste deel van het proefschrift beschouwd, want veel verder dan het dankwoord komt men vaak niet. As far as this thesis goes, I should start to thank Ana Achúcarro and Koenraad Schalm for allowing me to conduct my PhD research in their group, and for the encouragements and guidance they gave me. It has been a pleasure to work with you. Next, I would like to thank Kepa for helping me with the first steps in graduate research. Furthermore, Gonzalo Palma, Jinn-Ouk Gong and Subodh Patil, thank you for the pleasant cooperation which resulted in the work presented in chapters 4 and 5. Meer recent heb ik erg prettig samengewerkt met Ted van der Aalst en Johannes Oberreuter, wat een volgens mij goed artikel heeft opgeleverd. Daarnaast ook dank aan mijn leescommissie voor hun kritische blik.

The scientific endeavour can only flourish in a warm and friendly environment that the Intituut-Lorentz has offered me. I therefore would like to thank all my colleagues for distracting me way too long at the coffee table, institute trips and Christmas lunches. Nog een speciaal dankwoord voor mijn kamergenoot Louk Rademakers voor het tolereren van de bende die ik (af en toe) van onze kamer maakte. Ook wil ik het secretariaat bedanken voor het organiseren van deze goede sfeer.

Buiten het Instituut-Lorentz heb ik een erg prettig contact gehad met mijn collega's uit Amsterdam, Utrecht, Nijmegen en Groningen die ik af en toe op het Nikhef ontmoette. Daarnaast een speciaal dankwoord ook voor Johannes Oberreuter, voor het regelen van de koffie en toegang tot de bèta-faculteit van de UvA in de Watergraafsmeer.

Naast mijn studie en promotietraject heb ik veel te veel tijd gespendeerd in de roeiwereld. Een bijzonder dankwoord voor mijn eerstejaarsacht, *Understeightment* voor de steun die we elkaar tien jaar na dato nog steeds geven.

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## **Dankwoord**

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Figures 1.1 and 1.3, the figure on page 143 and the background of the letters on the cover are credited to the Legacy Archive for Microwave Background Data Analysis (LAMBDA). Support for LAMBDA is provided by the NASA Office of Space Science.

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# List of publications

1. Ana Achúcarro, Sjoerd Hardeman and Keba Sousa  
*Consistent Decoupling of Heavy Scalars and Moduli in  $N=1$  Supergravity*  
Phys. Rev. D78 101901 (2008), arXiv:0806.436 [Chapter 2]
2. Ana Achúcarro, Sjoerd Hardeman and Keba Sousa  
*F-term uplifting and the supersymmetric integration of heavy moduli*  
JHEP11:003 (2008), arXiv:0809.1441 [Chapter 3]
3. Sjoerd Hardeman  
*Quark matter influence on observational properties of compact stars*  
Nucl.Phys.A820:211C-214C (2009), arXiv:0810.2306
4. Ana Achúcarro, Jinn-Ouk Gong, Sjoerd Hardeman, Gonzalo A. Palma, Subodh P. Patil  
*Mass hierarchies and non-decoupling in multi-scalar field dynamics*  
arXiv:1005.3848 [Chapter 4]
5. Ana Achúcarro, Jinn-Ouk Gong, Sjoerd Hardeman, Gonzalo A. Palma, Subodh P. Patil  
*Features of heavy physics in the CMB power spectrum*  
JCAP01:030 (2011), arXiv:1010.3693 [Chapter 4&5]
6. Sjoerd Hardeman, Johannes Oberreuter, Gonzalo A. Palma, Koenraad Schalm and Ted van der Aalst  
*The everpresent  $\eta$ -problem: knowledge of all hidden sectors required*  
arXiv:1012.5966 [Chapter 1, section 3.2]



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# Curriculum Vitae

Ik ben op 27 januari 1982 geboren in Hengelo, Overijssel, Nederland. Nadat ik in Hengelo het basis- en voortgezet onderwijs heb gevolgd ben ik in 2000 naar Amsterdam gegaan om natuur- en sterrenkunde te gaan studeren. In 2005 heb ik de dubbele kandidaats natuur- en sterrenkunde behaald, in maart 2007 de masters natuurkunde en sterrenkunde. Naast het verplichte curriculum heb ik wat vakken aan de economische faculteit gevolgd en heb ik deelgenomen aan de “United Nations International Student Conference Amsterdam”, UNISCA. In mijn master heb ik in een groep onder leiding van Ralph Wijers deelgenomen aan de Academische Jaarprijs, een prijs voor het populariseren van toonaangevend wetenschappelijk onderzoek.

Na mijn studie heb ik een half jaar gewerkt als applicatiebeheerder bij Getronics, waar ik de webservers van de grote media onderhield. In november 2007 ben ik begonnen met mijn promotieonderzoek, wat hopelijk op één maart zal eindigen met het succesvol verdedigen van dit proefschrift.

Naast mijn studie heb ik me altijd veel met roeien beziggehouden. In 2000 zelf in de eerstejaars acht, in de jaren daarna als coach, kamprechter en in de organisatie van wedstrijden. Verder heb ik voor de Jongerenwerkgroep voor Sterrenkunde kampen begeleid en populair-wetenschappelijke voordrachten en teksten geleverd.