

# Improved action and Hamiltonian in finite volumes\*

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We introduce a new Symanzik improved action by adding a  $2 \times 2$  plaquette in such a way that the Feynman rules in the covariant gauge simplify. We call this the square Symanzik action. Some comparisons with the continuum and the standard Wilson action are made in intermediate volumes, where mass ratios are accurately known and the precise amount of improvement can be determined. Ratios of the Lambda parameters will be presented, as well as partial results for the one-loop improvement coefficients. We discuss some of the intricacies that arise because of violations of unitarity at the scale of the cutoff. In particular we show how a field redefinition in the zero-momentum effective action allows one to remove scaling violations linear in the lattice spacing.

## 1. INTRODUCTION

We consider here the Symanzik improvement scheme [1], which is designed to remove lattice artefacts by adding irrelevant operators to the lattice action, whose coefficients are tuned by requiring spectral quantities to be improved to the relevant order (on-shell improvement [2,3]). Perturbative calculations, although difficult, are still manageable. For Symanzik improvement to work it seemed that unreasonably small values of the bare coupling constant were required.

Mean field inspired Symanzik improvement [4, 5] was introduced to beat the bad convergence of perturbation expansions in the bare coupling constant. In particular the Parisi mean field coupling [4] defined in terms of the plaquette expectation value is seen to improve considerably the approach to asymptotic scaling. Despite some attempts [6] no good theoretical understanding for this is available. In addition the prescription is argued to include tadpole corrections to the coefficients in the Symanzik improved action, which can be seen as a mean field renormalization of the link variables on the lattice. Only phenomenological arguments have been provided to support this.

One difficulty in testing improvement is how

to determine to which extent improvement has actually been achieved. For pure gauge theories standard tests involve restoration of rotational invariance in the heavy quark potential [5]. It becomes more problematic when one has to base the judgement on carefully extrapolated Wilson data.

These problems inspired us to consider testing improvement for the pure gauge glueball spectrum in intermediate volumes, particularly emphasizing the need to test improvement of scaling. In spectroscopy asymptotic scaling is not such an important issue since one has to set the scale by fixing one of the masses anyhow. The main reason for considering intermediate volumes (up to 0.75 fermi across) is that this volume range can be accurately described in terms of an effective zero-momentum model, nevertheless incorporating important non-perturbative features that contribute to energy of electric flux. For SU(2) results are known both for the continuum limit and for the Wilson lattice action, from which precise statements on the scaling violations for the mass ratios can be made.

## 2. SQUARE SYMANZIK ACTION

As usual, one connects a continuum configuration with one on the lattice by parallel transport of the vector potential along the links

$$U_\mu(x) = P \exp\left(\int_0^a A_\mu(x + s\hat{\mu}) ds\right). \quad (1)$$

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This allows one to extract the irrelevant higher order operators that need to be cancelled in a lattice action. We introduce the new class of actions by adding a  $2 \times 2$  plaquette to the ones considered by Lüscher and Weisz,

$$\begin{aligned} S(\{c_i\}) &\equiv \sum_x \text{Tr}\{c_0 \langle 1 - \square \rangle + 2c_1 \langle 1 - \square \rangle + \\ &4c_2 \langle 1 - \square \rangle + 4c_3 \langle 1 - \square \rangle + c_4 \langle 1 - \square \rangle\} \\ &= -\frac{a^4}{2}(c_0 + 8c_1 + 8c_2 + 16c_3 + 16c_4) \sum_{x,\mu,\nu} \text{Tr}(F_{\mu\nu}^2(x)) + \\ &\frac{a^6}{12}(c_0 + 20c_1 - 4c_2 + 4c_3 + 64c_4) \sum_{x,\mu,\nu} \text{Tr}(\mathcal{D}_\mu F_{\mu\nu}(x))^2 + \\ &a^6 \left( \frac{c_2}{3} + c_3 \right) \sum_{x,\mu,\nu,\lambda} \text{Tr}(\mathcal{D}_\mu F_{\mu\lambda}(x) \mathcal{D}_\nu F_{\nu\lambda}(x)) + \\ &a^6 \frac{c_2}{3} \sum_{x,\mu,\nu,\lambda} \text{Tr}((\mathcal{D}_\mu F_{\nu\lambda})^2) + \mathcal{O}(a^8). \quad (2) \end{aligned}$$

Sometimes in the literature  $c_2$  and  $c_3$  are interchanged [7,8]. Here we followed the convention of ref. [2] and we have taken the liberty of assigning the coefficient  $c_4$  to the  $2 \times 2$  plaquette. The  $\langle \rangle$  imply summing  $\mu \neq \nu (\neq \lambda)$ , labelling the edges of the plaquette, with the point  $x$  attached to (say) the lower left corner [7]. At tree-level only the planar loops are considered,  $c_2$  and  $c_3$  acquire non-zero values only at one-loop order, but as was shown by Lüscher and Weisz [3] field redefinitions allow one to put  $c_3 \equiv 0$ . For the LW Symanzik action ( $c_4 = 0$ ) one has  $c_0 = 5/3$  and  $c_1 = -1/12$  at tree-level, but this does not allow for a “covariant” gauge condition that will make the gauge field propagator diagonal in the space-time indices. The  $2 \times 2$  plaquette allows one to “complete a square” when choosing  $c_4 \cdot c_0 = c_1^2$ , leading to the gauge fixing functional ( $z \equiv c_1/c_0$ )

$$\mathcal{F}_{gf} \equiv \sqrt{c_0} \sum_\mu \partial_\mu^\dagger (1 + z(2 + \partial_\mu^\dagger)(2 + \partial_\mu)) q_\mu(x). \quad (3)$$

We decided for this reason to call it the square Symanzik action. Here  $\partial_\mu$  denotes the lattice difference operator  $\partial_\mu \varphi(x) \equiv \varphi(x + \hat{\mu}) - \varphi(x)$ . As a bonus we note that the condition  $c_4 c_0 = c_1^2$  is invariant under multiplicative link renormalization, as they appear in the tadpole improvement

scheme, allowing one to easily include such factors in a perturbative calculation. At tree-level one now finds  $c_0 = 16/9$ ,  $c_1 = -1/9$  and  $c_4 = 1/144$ . We have verified that this action satisfies the positivity bound [3]. It is amusing to see the expression for the  $a^8$  term in the expansion of the action simplify to

$$S = \sum_{x,\mu,\nu} \text{Tr} \left[ \frac{a^4}{2} F_{\mu\nu}^2(x) - \frac{a^8}{90} (\mathcal{D}_\mu^2 F_{\mu\nu}(x))^2 \right] + \mathcal{O}(a^{10}) \quad (4)$$

(behaving as the Symanzik action for cooling [7]).

At tree-level a tadpole parameter  $u_0$  modifies  $z = c_1/c_0 = -1/16$  to  $z = -1/16u_0^2$ , whereas  $c_0 = 1/(1+4z)^2$ . One finds in the covariant gauge the ghost ( $P$ ) and vector ( $P_{\mu\nu}$ ) propagators to be

$$\begin{aligned} P(k) &= \frac{1}{\sqrt{c_0} \sum_\lambda (4 \sin^2(k_\lambda/2) + 4z \sin^2 k_\lambda)}, \\ P_{\mu\nu}(k) &= \frac{P(k) \delta_{\mu\nu}}{\sqrt{c_0} (1 + 4z \cos^2(k_\mu/2))}. \quad (5) \end{aligned}$$

### 3. EFFECTIVE ACTION

One can now perform a background field calculation to determine the one-loop effective action for the zero-momentum gauge fields. For the Wilson action and the continuum this was done previously [9] and shown to lead to rather accurate results. Like for the Wilson case one writes  $U_\mu(x) = e^{\hat{q}_\mu(x)} e^{c_\mu(t)/N}$ , with  $N$  the number of lattice sites in the spatial direction (taking the number of sites in the time direction infinite) and  $\hat{q}_\mu(x)$  the quantum field, restricted to non-zero (spatial) momentum, to be integrated out. This choice on splitting off the quantum component of the lattice field yields a particularly simple background gauge fixing function

$$\hat{\mathcal{F}}_{gf} \equiv \sqrt{c_0} \sum_\mu \hat{D}_\mu^\dagger (1 + z(2 + \hat{D}_\mu^\dagger)(2 + \hat{D}_\mu)) \hat{q}_\mu(x), \quad (6)$$

“covariantizing” the difference operator to

$$\hat{D}_\mu \varphi(x) \equiv e^{c_\mu(t)/N} \varphi(x + \hat{\mu}) e^{-c_\mu(t)/N} - \varphi(x). \quad (7)$$

#### 3.1. The SU(2) effective potential

The one-loop calculation greatly simplifies for an abelian constant background field as this allows one to diagonalize the propagator with respect to the isospin neutral and charged decomposition of the gauge and ghost fields. The momenta

in the background field  $\vec{c} = \frac{1}{2}i\vec{C}\sigma_3$  are shifted,  $\vec{k} \rightarrow \vec{k} + s\vec{C}/N$ , where  $s = 0$  for the neutral isospin component and  $s = \pm 1$  for the two charged components. It is not hard to find the eigenvalues of the fluctuation operators for the ghost and gauge fields

$$\begin{aligned}\lambda_{gh}(k) &= \sqrt{c_0} \sum_{\nu} 4 \sin^2(k_{\nu}/2)(1+4z \cos^2(k_{\nu}/2)), \\ \lambda_{\mu}(k) &= \sqrt{c_0}(1+4z \cos^2(k_{\mu}/2))\lambda_{gh}(k).\end{aligned}\quad (8)$$

These eigenvalues can be written as products of factors  $4 \sin^2(k_0/2) + \omega_{\alpha}^2$ , where the  $\omega_{\alpha}$  can occur in complex conjugate pairs at spatial momenta close to the edge of the Brillouin zone. As it is well-known that the sum over  $k_0$  for one such a factor can be performed explicitly, it is not surprising we can find a closed expression for the effective potential, as a sum over the appropriately weighted logarithm of the eigenvalues

$$\begin{aligned}V_1^{ab}(\vec{C}) &= N \sum_{\vec{n} \in \mathbb{Z}_N^3} \left\{ \sum_i \log \left( 1 + 4z \cos^2 \left[ \frac{2\pi n_i + C_i}{2N} \right] \right) \right. \\ &\quad \left. + 4 \operatorname{asinh} \left( 2u_0 \sqrt{1 + 4z + \frac{\omega^2}{2}} + \omega \sqrt{1 + \frac{\omega^2}{4}} \right) \right\},\end{aligned}\quad (9)$$

with  $\omega^2 \equiv 4 \sum_i \sin^2(k_i/2)(1+4z \cos^2(k_i/2))$ , and  $\vec{k} = (2\pi\vec{n} + \vec{C})/N$ . This effective potential, normalized to  $V(\vec{0}) = 0$ , is plotted for  $u_0 = 1$  in fig. 1 as compared to the result for the Wilson action ( $z \equiv 0$ ) and for the continuum ( $N \rightarrow \infty$ ). Although this effective potential is not spectral, since near  $\vec{C} \in 2\pi\mathbb{Z}^3$  the adiabatic approximation for integrating out the “charged” zero-momentum modes breaks down, one sees that improvement is quite efficient in removing scaling violations (only scaling violations to fourth order in the lattice spacing  $a = 1/N$  remain). At  $N = 6$  we can not distinguish the result from the continuum at the scale of this figure. One might even fear that choosing  $u_0 \neq 1$  makes the agreement worse.

### 3.2. The Lambda ratios

One can proceed as in the Wilson case with computing the one-loop coefficients for the effective action [9]. In particular this provides for  $N \rightarrow \infty$  the renormalization of the bare lattice

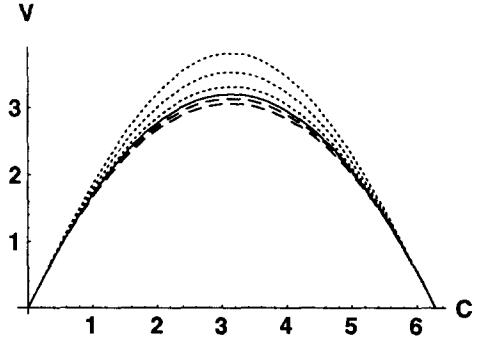


Figure 1. The effective potential for a constant Abelian background field  $c_1 = \frac{1}{2}iC\sigma_3$ . The full line represents the continuum result. The lower two dashed curves are for the square Symanzik action with  $N = 3$  and  $4$ . The upper three dotted curves are for the Wilson action with  $N = 3, 4$  and  $6$ .

coupling at fixed physical volume. As Lorentz invariance is replaced by cubic invariance (both by the lattice discretization and the periodic boundary conditions), two independent determinations of the Lambda ratios can be extracted from the effective Lagrangian

$$\frac{1}{2} \left( \frac{1}{g^2} + \alpha_1 \right) \left( \frac{dc_i^a}{dt} \right)^2 + \frac{1}{4} \left( \frac{1}{g^2} + \alpha_2 \right) (F_{ij}^a)^2 + V_1(c). \quad (10)$$

Here  $g^{-2} = g_0^{-2} - 11 \log(N)/12\pi^2$  is kept fixed while taking the continuum limit. In this limit  $\alpha_1$  and  $\alpha_2$  differ by *identical* finite amounts from what was found for the continuum and the Wilson action. This finite difference allows one to accurately compute the Lambda parameter ratios. These ratios were also computed using the heavy quark potential method of ref. [8], which allows one in addition to obtain the result for SU(3). For the ratio of the square Symanzik action to the Wilson action we find.

$$\Lambda_{S^2}/\Lambda_W = \begin{cases} 4.0919901(1) & \text{for SU(2)} \\ 5.2089503(1) & \text{for SU(3)} \end{cases}. \quad (11)$$

### 3.3. One-loop improvement

Tests of tadpole corrections to variant tree-level improved actions have been performed before [10]. However, we remind the reader that

there is only one pure gauge improved lattice action that was computed to one-loop order [2]. It is our aim to bring the square Symanzik action to this same level. In principle this provides a way of testing to what extent the success of the tadpole improvement depends on the choice of action. Independently it is a useful check on the consistency of the Symanzik improvement scheme with its inherent redundancy in choosing the lattice action to cancel scaling violations. For these perturbative calculations of the one-loop corrections  $c'_i$  one follows the well established route of using the twisted finite volume spectroscopy [2]. As a normalization condition on the definition of the coupling constant one imposes  $c'_0 + 8c'_1 + 8c'_2 + 16c'_4 = 0$ . Requiring the physical mass of the lowest state to have no quadratic scaling violations to one-loop order for the square Symanzik action leads to

$$c'_1 - c'_2 + 4c'_4 = \begin{cases} -0.00838(1) & \text{for SU}(2) \\ -0.01545(2) & \text{for SU}(3) \end{cases}. \quad (12)$$

As an independent check this combination was also extracted (at higher accuracy) from the heavy quark potential. In addition the on-shell three point coupling extracted in the twisted finite volume allows one to find [2] a value for  $36(c'_1 - c'_2 + 4c'_4) + 8c'_2$ . This computation is rather involved and still in progress. Note that  $c'_4$  appears in the combinations  $c'_1 + 4c'_4$  and  $c'_0 - 16c'_4$ , as is also dictated by eq. (2). Therefore as was to be expected  $c'_4$  is a free parameter. It *need not*, but can, be fixed by requiring  $c_4 c_0 = c_1^2$  to one-loop order. Finally we quote the result for the single plaquette expectation value:  $u_0^4 = 1 - 0.35878 \cdot g_0^2(N - N^{-1})/4$ .

#### 4. MONTE CARLO RESULTS

In the intermediate volume context we can consider at most lattice spacings up to 0.25 fermi (asking to be absolved for pushing in this direction), as the larger plaquettes that appear in the improved actions require the volume to be at least three lattice spacings in each direction. Despite the appearance of unitarity violations at the scale of the cutoff, due to the non-local nature of an improved action, the intermediate volume physical

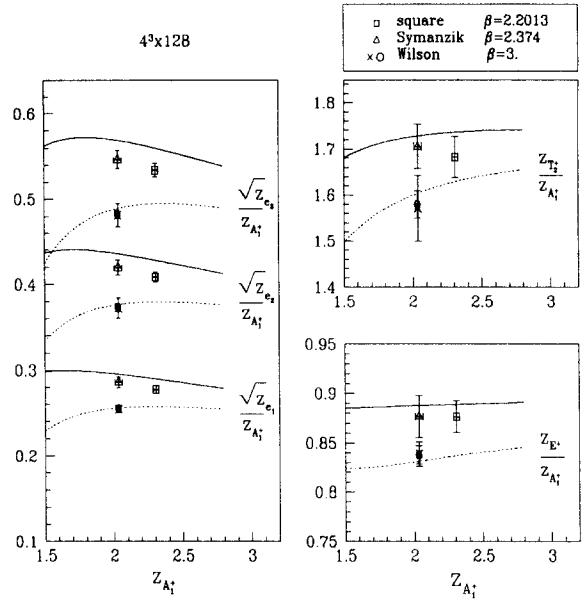


Figure 2. SU(2) Monte Carlo data for mass ratios in a small volume on a lattice of size  $4^3 \times 128$ , using the Wilson action (crosses from existing data of Michael), the LW Symanzik action and our new square Symanzik action. The lines give the analytic results: full for the continuum and dotted for the standard lattice action ( $N = 4$ ).

masses remain small enough in lattice units to extract them from the decay of correlation functions in the time direction in the usual way. In larger volumes and coarser lattices the latter problem was dealt with by taking the lattice much finer in the time direction [11], using the asymmetric couplings well-known from finite temperature studies [12]. Here we will consider only Monte Carlo data at 0.018 fermi ( $\beta = 4/g_0^2 = 3$  for the Wilson action, for which we compare our data to those by Michael [13]). Odd as it may seem, this is where the scaling violations for a lattice of 4 lattice spacings in the spatial directions are largest within the finite volume spectroscopy [9]. The data corresponding to the LW Symanzik action is represented by the triangles and for our new square Symanzik action by the squares. In both cases we used tree-level improvement only. The improvement is significant. For the LW Symanzik

action the data is within two sigma of the continuum values. The results seem to indicate that the square Symanzik action is somewhat less effective, although the difference is not significant. A comparison at coarser lattices will be more interesting as one should like to see, as advocated, tadpole corrections to further improve the results. For this purpose we present data elsewhere at a lattice spacing of 0.12 fermi.

## 5. HAMILTONIAN

### 5.1. Toy model

A well-known problem of improved actions is that the transfer matrix is not hermitian [14]. This is easily seen to be related to the next-to-nearest neighbor couplings in the time direction. We will illustrate things here by means of a simple one dimensional problem. For the action  $S(x) = \sum_t (x(t+1) - x(t))^2/(2g^2a) + aV(x(t))$  the partition function at finite Euclidean time ( $T = aN$ ) can be exactly rewritten in operator form [9]

$$\mathcal{Z} = \int \mathcal{D}x e^{-S(x)} = \text{Tr}(e^{-\frac{1}{2}aK} e^{-aV} e^{-\frac{1}{2}aK})^N, \quad (13)$$

where  $K = -\frac{1}{2}g^2(\partial/\partial x)^2$ . The Hamiltonian read off from this equation is only determined up to a unitary transformation. To lowest order one finds

$$H = K + V - \frac{a^2}{24}[V, [K, V]] + \mathcal{O}(a^4). \quad (14)$$

Note that  $[V, [K, V]] = g^2V'(x)^2$ .

Next improve the kinetic term  $(x(t+1) - x(t))^2$  by  $4(x(t+1) - x(t))^2/3 - (x(t+2) - x(t))^2/12$ . One finds that the propagator factorizes as  $P(k) = (P_-(k)P_+(k))/Z$ , where  $P_{\pm}^{-1}(k) = 4\sin^2(\frac{1}{2}k) + \omega_{\pm}^2$ ,  $Z = \sqrt{1-a^2m^2/3}$ ,  $\omega_{\pm}^2 = 6(1 \pm Z)$  and  $m^2 = g^2V''(0)$ . This explicitly exhibits the unphysical pole mentioned before with masses  $m_{\pm}^2 \sim 12/a$  at the scale of the cutoff. They are not harmful for low-energy behavior [14]. It would perhaps be misleading to associate the spurious poles with ghosts as they do not just occur in loops. Vertices do not preserve ghost number. Nevertheless we expect their contribution to low-lying states to be suppressed in a way similar to the influence of virtual processes due to heavy particles.

Let us introduce the following field redefinition, best expressed in the Fourier representation

$$\bar{x}(k) = x(k)\sqrt{1+\frac{1}{12}\hat{k}^2} - \frac{a^2g^2}{24}\frac{\partial V(\bar{x})}{\partial \bar{x}(-k)}, \quad (15)$$

where as usual  $\hat{k} = 2\sin(k/2)$ . When substituting this non-local transformation in the action we find

$$\begin{aligned} S &= \sum_k \frac{\hat{k}^2(1+\frac{1}{12}\hat{k}^2)|x(k)|^2}{2ag^2} + aV(x) \\ &= \sum_k \frac{\hat{k}^2|\bar{x}(k)|^2}{2ag^2} + aV(\bar{x}) + \frac{a^3g^2}{24}V'(\bar{x})^2 + \mathcal{O}(a^5), \end{aligned} \quad (16)$$

for which it is assumed that  $\hat{k} = \mathcal{O}(a)$ . We note that after the field redefinition, ignoring the  $\mathcal{O}(a^5)$  corrections, the action is local in time and one obtains  $H = K + V + \mathcal{O}(a^4)$  from eq. (14).

However, interactions will give rise to a non-trivial Jacobian under this change of variables,  $J(x) = \det(\partial\bar{x}(k)/\partial x(k'))$ , or

$$J^2(x) = \det\left(1 + \frac{1}{12}\partial_0^\dagger\partial_0 - \frac{1}{12}a^2g^2\frac{\partial^2 V(x)}{\partial x^2}\right), \quad (17)$$

where we took the liberty of modifying the  $\mathcal{O}(a^4)$  terms in the operator whose determinant is to be evaluated. We could likewise define the transformation such that the Jacobian is given as above, although this leads in multi-dimensional cases to non-integrable transformations.

Remarkably one can rewrite this Jacobian up to an  $x$  independent factor as

$$J^2(x) = \det\left(1 - a^2g^2P_0\left[\frac{\partial^2 V(x)}{\partial x^2} - m^2\right]\right). \quad (18)$$

Its contribution to the partition function  $\mathcal{Z}$  can be interpreted as the effective action in a background field calculation, with the propagator truncated to the unphysical branch, albeit to lowest non-trivial order in the lattice spacing. One easily verifies that for  $V(x) = \lambda x^4$  this Jacobian gives rise to a mass correction *linear* in the lattice spacing, which was initially discovered by computing the mass gap to first order in  $\lambda$  from the Feynman rules for the improved action of this simple model. As the model is quite similar to the effective action we discussed before, we were quite puzzled by this result and it prompted the above derivation.

## 5.2. Gauge model

Indeed, taking the results of sect. 3.1 we can compute easily part of the effective action for the zero-momentum gauge fields. To obtain the effective potential that is valid near  $\tilde{C} = \vec{0}$ , where the tree-level potential is quartic in the gauge fields, one restricts the sum to  $\vec{n} \neq \vec{0}$  and replaces  $C_i$  by  $r_i \equiv \sqrt{-2\text{Tr}c_i^2}$  in eq. (9). The result is denoted by  $\hat{V}_1(\vec{r})$ . An accurate description of the full effective potential to one-loop order is given by

$$V_1(c) = \hat{V}_1(\vec{r}) + \alpha_3 r_i^2 F_{jk}^{a^2} + \alpha_4 r_i^2 F_{ij}^{a^2} + \alpha_5 \det^2 c \quad (19)$$

As can be extracted from the zero-momentum part of eq. (9),  $\gamma_1(N) = \gamma_1(\infty) + \frac{1}{12}(\sqrt{3}-1)/N + \mathcal{O}(1/N^3)$ , where  $\gamma_1$  is the coefficient of  $\vec{r}^2$  in the effective potential. Generalizing the analysis of the toy model to the situation at hand one finds from the Jacobian  $\delta_J \gamma_1 = -\frac{1}{12}\sqrt{3}/N$ . The missing piece is provided by the non-triviality of the Haar measure for integration over the background link variables

$$\delta_H V_1(c) = -2N \sum_i \log[2N \sin(r_i/2N)/r_i], \quad (20)$$

Indeed one finds  $\delta_H \gamma_1 = \frac{1}{12}N^{-1}$ . Both the Jacobian and measure contributions compensate for the scaling violations linear in the lattice spacing and with it  $\hat{V}_1(\vec{r})$  becomes free of scaling violations to third order in the lattice spacing.

Furthermore, rescaling  $c$  with  $(1 + \frac{1}{12}g^2 \gamma_1/N^2)$  removes to a high degree of accuracy unwanted scaling violations in  $\alpha_1$  and  $\alpha_2$ . More surprising was to find that the field redefinition  $\delta c_i = -2g^2 \log(N) D_\mu^\dagger F_{\mu i} / (24\pi N)^2$  is required to remove  $\log(N)/N^2$  scaling violations in  $\alpha_3$  and  $\alpha_5$ . As a non-trivial check the one-loop coefficient  $\alpha_0$ , in front of the term  $\frac{1}{2}(\partial_0^\dagger \partial_0 c_i^a)^2$ , was computed. Its  $\log(N)/N^2$  term combines after the above field redefinition with the tree-level coefficient of  $1/(12g_0^2 N^2)$  in precisely the right way to renormalize the coupling constant.

Remaining  $\mathcal{O}(N^{-2})$  scaling violations will and can be cancelled by the one-loop improvement coefficients. Due to the unfortunate mixing with coefficients that are not easily accessible in lattice perturbation theory [9], we cannot at present get at these one-loop improvement coefficients along

this route. As we have seen, using an effective action one imposes improvement only up to field redefinitions (or up to unitary transformations at the Hamiltonian level). This is more difficult than computing a few spectral quantities, but has the obvious benefit of manifestly improving infinitely many levels at the same time and would be a very non-trivial check on improvement indeed.

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