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A Monte Carlo study of old, new and tadpole improved actions

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Abstract

Scaling of mass ratios in intermediate volumes, obtained with improved SU(2) lattice actions is tested against analytic results for the Wilson and continuum action. A new improved action is introduced by adding a 2×2 plaquette to the Symanzik action. Completing a square leads to a covariant propagator that simplifies perturbative calculations. Data is presented on lattices of size $4^3 \times 128$, with lattice spacings of approximately 0.02 and 0.12 fermi. For the latter case no further improvement as compared to the tree-level action was observed when including the Lepage-Mackenzie tadpole correction to the one-loop improved Lüscher-Weisz Symanzik action.

1. Introduction

Improvement of lattice actions aims at doing Monte Carlo simulations on coarser lattices, such that with a modest number of lattice spacings the physical volume is sufficiently large. But perhaps more importantly it should make extrapolations to the continuum limit more reliable, as has been one of the main objectives in the non-perturbative determination of the running coupling constant [1]. Here we consider the Symanzik improvement scheme [2], which is designed to remove lattice artefacts by adding irrelevant operators to the lattice action, whose coefficients are tuned by requiring spectral quantities to be improved to the relevant order (on-shell improvement [3]). For Symanzik improvement to work it seemed that unreasonably small values of the bare coupling constant were required.

Mean field inspired Symanzik improvement [4] was introduced to beat the bad convergence of perturbative expansions in the bare coupling constant. In particular the Parisi mean field coupling [5] defined in terms of the plaquette expectation value is seen

to improve considerably the approach to asymptotic scaling. Despite some attempts [6] no good theoretical understanding for this is available. The tadpole prescription also includes corrections [4] to the coefficients in the Symanzik improved action, which can be seen as a mean field renormalization of the link variables on the lattice. Only phenomenological arguments have been provided to support this. Standard tests in pure gauge theories involve restoration of rotational invariance [4]. More involved, but of direct physical relevance, are the tests in charmonium spectroscopy [7], used to extract a value of the strong coupling constant [8].

We here stress the necessity of improving scaling, rather than asymptotic scaling, which in spectroscopy is less important since one has to set the scale by one of the masses or the string tension. Our study is complementary to that of Ref. [9]. Although one is ultimately interested in the infinite volume limit, from the point of studying the approach to the continuum limit a finite volume provides a useful tool. If improvement fails there, it sheds doubt on results in large volumes (when successful, however, one does not imply the other).

Perhaps a somewhat inappropriate comparison is that we consider our study as a well controlled laboratory experiment, where conditions are manipulated so as to rule out as much as possible external disturbances.

The setup of this letter is to first introduce and motivate the new improved action. It simplifies certain perturbative calculations, and provides in part an analytic test of improvement in a small volume for which we present the effective potential in a constant abelian background field. Also the Lambda parameter of the new improved action is related to that of the Wilson action. We then present our Monte Carlo data at very small and intermediate volumes and end with conclusions. Details of the analytic study will be presented elsewhere (preliminary material and some further discussion can also be found in two communications to conferences [10]).

2. Square Symanzik action

There is a large redundancy in choosing an improved action, when parametrized in terms of Wilson loops. We shall use this to allow for a simplified “covariant” gauge choice, achieved by adding to the Lüscher-Weisz (LW) Symanzik action a 2×2 plaquette:

$$\begin{aligned}
 S(\{c_i\}) \equiv & \sum_x \text{Tr} \left\{ c_0 \sum_{\mu \neq \nu} \left\langle 1 - \nu \begin{array}{c} \square \\ \mu \end{array} \right\rangle \right. \\
 & + 2c_1 \sum_{\mu \neq \nu} \left\langle 1 - \nu \begin{array}{c} \square \\ \mu \end{array} \right\rangle \\
 & + \frac{4}{3}c_2 \sum_{\mu \neq \nu \neq \lambda} \left\langle 1 - \nu \begin{array}{c} \square \\ \mu \end{array} \right\rangle \\
 & + 4c_3 \sum_{\mu \neq \nu \neq \lambda} \left\langle 1 - \nu \begin{array}{c} \square \\ \mu \end{array} \right\rangle \\
 & \left. + c_4 \sum_{\mu \neq \nu} \left\langle 1 - \nu \begin{array}{c} \square \\ \mu \end{array} \right\rangle \right\}. \tag{1}
 \end{aligned}$$

The $\langle \rangle$ imply averaging over the two opposite directions for each of the links, called “clover” averaging in Ref. [11]. Numerical factors were chosen to agree with earlier conventions [3]. Note that sometimes c_2 and c_3 are interchanged in the literature [11,12]. Here c_4 is assigned to the 2×2 plaquette.

The number of parameters required to improve the action to a certain order is simply determined from the number of gauge and hypercubic invariant operators that one can write down up to that order (read off from the dimension of the operator). For pure gauge theories there is only one operator of dimension zero and three of dimension two. One of these is redundant as it can be removed by a field redefinition, which can also be implemented at the level of the Wilson loop representation. It allows one to choose [3] $c_3 = 0$.

As usual we relate lattice and continuum fields by $U_\mu(x) = P \exp(\int_0^a A_\mu(x + s\hat{\mu}) ds)$. This gives the following expansion for the lattice action [11,12]:

$$\begin{aligned}
 S(\{c_i\}) = & -\frac{a^4}{2}(c_0 + 8c_1 + 8c_2 + 16c_3 + 16c_4) \\
 & \times \sum_{x,\mu,\nu} \text{Tr}(F_{\mu\nu}^2(x)) \\
 & + a^6 \left(\frac{c_2}{3} + c_3\right) \sum_{x,\mu,\nu,\lambda} \text{Tr}(\mathcal{D}_\mu F_{\mu\lambda}(x) \mathcal{D}_\nu F_{\nu\lambda}(x)) \\
 & + a^6 \frac{c_2}{3} \sum_{x,\mu,\nu,\lambda} \text{Tr}((\mathcal{D}_\mu F_{\nu\lambda}(x))^2) \\
 & + \frac{a^6}{12}(c_0 + 20c_1 - 4c_2 + 4c_3 + 64c_4) \\
 & \times \sum_{x,\mu,\nu} \text{Tr}(\mathcal{D}_\mu F_{\mu\nu}(x))^2 + \mathcal{O}(a^8). \tag{2}
 \end{aligned}$$

To fix the definition of the coupling constant one imposes $(c_0 + 8c_1 + 8c_2 + 16c_3 + 16c_4) = 1$. Computing two particular spectral quantities as a function of these parameters allows one to determine these coefficients. At tree-level the conventional choice amounts to putting $c_0 = 5/3$, $c_1 = -1/12$ and $c_2 = c_3 = c_4 = 0$. The one-loop ($\mathcal{O}(g_0^2)$) correction to these coefficients was computed by Lüscher and Weisz [3]. For $c_4 \neq 0$ a similar calculation is in the process of being completed by one of us. At tree-level we have fixed c_4 by the following requirement. When expanding the action to quadratic order in the lattice field $q_\mu(x)$, defined by $U_\mu(x) = \exp(q_\mu(x))$, one finds

$$\begin{aligned}
 S_2 = & \sum_{x,\mu,\nu} -\frac{1}{2} \text{Tr} [c_0 (\partial_\mu q_\nu(x) - \partial_\nu q_\mu(x))^2 \\
 & + 2c_1 \{ (2 + \partial_\mu) (\partial_\mu q_\nu(x) - \partial_\nu q_\mu(x)) \}^2 \\
 & + c_4 \{ (2 + \partial_\nu) (2 + \partial_\mu) (\partial_\mu q_\nu(x) - \partial_\nu q_\mu(x)) \}^2], \tag{3}
 \end{aligned}$$

where ∂_μ is the lattice difference operator $\partial_\mu \varphi(x) \equiv \varphi(x + \hat{\mu}) - \varphi(x)$. If we now choose

$$c_4 \equiv z^2 c_0, \quad z \equiv c_1/c_0, \quad (4)$$

we can complete squares and obtain a simple gauge fixing function

$$\mathcal{F}_{gf}(x) \equiv \sqrt{c_0} \sum_\mu \partial_\mu^\dagger (1 + z(2 + \partial_\mu^\dagger)(2 + \partial_\mu)) \times q_\mu(x). \quad (5)$$

It is for this reason we propose to call the new improved action the square Symanzik action. At tree-level one finds

$$c_0 = 16/9, \quad c_1 = -1/9, \quad c_2 = 0, \\ c_3 = 0, \quad c_4 = 1/144. \quad (6)$$

An amusing, and potentially useful feature is that the relation $c_4 c_0 = c_1^2$ is not affected by tadpole corrections, where one replaces $U_\mu(x)$ by $U_\mu(x)/u_0$, with u_0 the fourth root of the average value of the plaquette:

$$u_0^4 = \text{Re } k^{-1} \text{Tr} \left\langle \begin{array}{|c|} \hline \square \\ \hline \end{array} \right\rangle. \quad (7)$$

Here k is the number of colors. For values of $u_0 \neq 1$ one easily finds $z = -1/(16u_0^2)$ and $c_0 = 1/(1+4z)^2$.

In the covariant gauge the propagators for the ghost and vector fields are simply given by

Ghost :

$$P(k) = \frac{1}{\sqrt{c_0} \sum_\lambda (4 \sin^2(k_\lambda/2) + 4z \sin^2 k_\lambda)},$$

$$\text{Vector : } P_{\mu\nu}(k) = \frac{P(k) \delta_{\mu\nu}}{\sqrt{c_0} (1 + 4z \cos^2(k_\mu/2))}. \quad (8)$$

It illustrates an important feature of improved actions with more than nearest neighbor couplings in the time direction: unphysical poles appear at the scale of the cutoff, $1/a$. For low energy physics they are harmless [13] in the same way Pauli-Villars regulator fields are harmless at energies below the scale of the cutoff. However, on the lattice these spurious poles are more cumbersome to handle as they do not simply appear in loops (i.e. vertices do not preserve something like ghost number). Nevertheless, there is a way of separating off their contributions [10]. Each of the propagators can be factorized in the sum of two or

three standard (single pole) lattice propagators, $P_s \equiv 1/(4 \sin^2(\frac{1}{2}k_0) + \omega_s^2(\mathbf{k}))$,

$$P(k) = Z(\mathbf{k})(P_-(k) - P_+(k)),$$

$$P_{\mu\nu}(k) = \delta_{\mu\nu}(Z_\mu^-(\mathbf{k})P_-(k) - Z_\mu^+(\mathbf{k})P_+(k) + Z_\mu^0(\mathbf{k})P_0(k)),$$

It is straightforward to derive the explicit expressions for the Z factors and energies ω from Eq. (8). Note that $Z_j^0 = 0$ and that (for $u_0 = 1$) $\omega_+^2(\mathbf{0}) = \omega_-^2(\mathbf{0}) = 12$. The spurious poles in this case occur at an energy $2 \text{asinh}(\sqrt{3})/a$.

One particularly simple test of improvement is achieved by computing for $SU(2)$ the effective potential for a static abelian zero-momentum background field, $\hat{U}_j = \exp(\frac{1}{2}iC_j \sigma_3/N)$ and $\hat{U}_0 = 1$, that is a solution of the (lattice) equations of motion. Introducing the quantum fluctuations through $U_\mu(x) = e^{\hat{q}_\mu(x)} \hat{U}_\mu$ one easily diagonalizes the quadratic fluctuation operator in the covariant gauge

$$\hat{\mathcal{F}}_{gf} \equiv \sqrt{c_0} \sum_\mu \hat{D}_\mu^\dagger (1 + z(2 + \hat{D}_\mu^\dagger)(2 + \hat{D}_\mu)) \times \hat{q}_\mu(x), \quad (9)$$

where $\hat{D}_\mu \varphi(x) \equiv \hat{U}_\mu \varphi(x + \hat{\mu}) \hat{U}_\mu^\dagger - \varphi(x)$. Due to the background field, momenta will be shifted to $k^s = (2\pi n + sC)/N$, where $s = 0$ for the isospin neutral and $s = \pm 1$ for the isospin charged components of quantum fields. The eigenvalues can be directly read off from Eq. (8) and one finds

$$V_1^{\text{ab}}(C) = N \sum_{n \in \mathbb{Z}_N^3} \left\{ \sum_i \log(\lambda_i) + 4 \text{asinh} \left(2u_0 \sqrt{1 + 4z + \frac{\omega^2}{2} + \omega \sqrt{1 + \frac{\omega^2}{4}}} \right) \right\}, \quad (10)$$

with $\lambda_j = 1 + 4z \cos^2(\frac{1}{2}k_j^+)$ and $\omega^2 = \sum_j 4\lambda_j \times \sin^2(\frac{1}{2}k_j^+)$. The result, normalized to $V_1^{\text{ab}}(\mathbf{0}) = 0$, is plotted in Fig. 1 for $u_0 = 1$ (together with the effective potential for the Wilson action, obtained by taking $z = 0$). At $N = 6$ we can not distinguish the result from the continuum at the scale of this figure.

We can use the abelian background field also to compute the one-loop correction to the tree-level kinetic term $\frac{1}{2}g_0^{-2}(dC_i(t)/dt)^2$, which yields

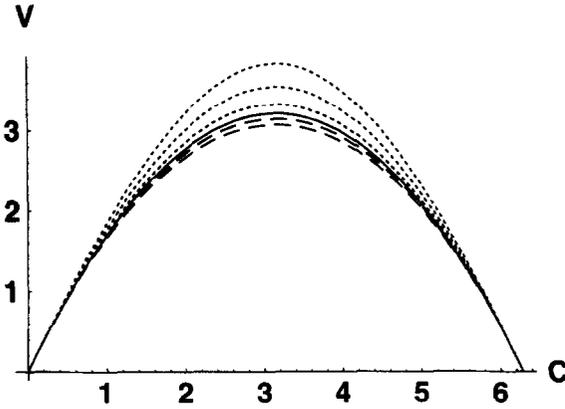


Fig. 1. The SU(2) effective potential for a constant abelian background field $C = (C, 0, 0)$. The full line represents the continuum result (obtained by taking the number of lattice spacings $N \rightarrow \infty$). The lower two dashed curves are for the square Symanzik action ($\mu_0 = 1$) with $N = 3$ and 4 . The upper three dotted curves are for the Wilson action with $N = 3, 4$ and 6 .

$\frac{1}{2}(g^{-2} + \alpha_1)(dC_i(t)/dt)^2$. In the continuum limit $g^{-2} = g_0^{-2} - 11 \log(N)/12\pi^2$ is kept fixed while sending the number of lattice spacings, N , to infinity. An analytic expression for $\alpha_1(N)$ was found in terms of a sum over spatial momenta, which reduces to the result for the Wilson action [14] at $z = 0$. Computing this sum for one hundred lattices and fitting the result to a polynomial in $1/N$ we find $\alpha_1 = -0.0340012235(1)$ for $z = -1/16$ and $\alpha_1 = -0.1648688946(1)$ for $z = 0$. From the difference one determines the ratios of the Lambda parameters between the square Symanzik (Λ_{S^2}) and the Wilson (Λ_W) actions. One can also compute the one-loop correction for $\frac{1}{4}g_0^{-2} \text{Tr} F_{ij}^2$, which gives an identical result for the Lambda ratios. Alternatively, we used the heavy quark potential method [12], which also allowed us to extract the Lambda ratios for SU(3). We quote the following result:

$$\begin{aligned} \Lambda_{S^2}/\Lambda_W[\text{SU}(2)] &= 4.0919901(1), \\ \Lambda_{S^2}/\Lambda_W[\text{SU}(3)] &= 5.2089503(1). \end{aligned} \quad (11)$$

In addition, the one-loop perturbative expansions of the SU(k) expectation values for an $a \times b$ plaquette, $U(P(a, b))$, are given by

$$\langle \text{Re} k^{-1} \text{Tr} U(P(a, b)) \rangle = 1 - \frac{1}{4}g_0^2(k - k^{-1})w(a, b). \quad (12)$$

For the square Symanzik action (Eq. (6)) we find

$$\begin{aligned} w(1, 1) &= 0.3587838551(1), \\ w(1, 2) &= 0.6542934512(1), \\ w(2, 2) &= 1.0887235337(1). \end{aligned} \quad (13)$$

3. Monte Carlo data

We wish to determine in small volumes the mass for the scalar (A_1^+) and tensor glueballs, the latter split due to the breaking of rotational invariance in the doublet E^+ and the triplet T_2^+ . Also the energies of the electric flux (“torelon”) states with one, two and three units of electric flux (e_i , $i = 1, 2, 3$) will be measured. In addition we consider the states with two (T_{11}^+ or $B(110)$) and three ($T_2^+(111)$) units of electric flux that have T_2^+ quantum numbers (negative parity in two directions of electric flux, symmetrized in those two directions). See Ref. [15] for details and further references.

The size of the lattice used is $4^3 \times 128$ and masses m are converted to dimensionless parameters into $z = mL$; in lattice units we hence multiply the mass with the number of lattice sites in the spatial directions. In large volumes one should have $z_{ek} = \sigma L^2 \sqrt{k}$, where σ is the infinite volume string tension. This is why we will consider the ratios $\sqrt{z_{ek}}/z_{A_1^+}$. These and other mass ratios will be plotted as a function of $z_{A_1^+}$. The analytic result [14] derived by diagonalizing an effective Hamiltonian to describe low-lying states is valid up to $z_{A_1^+} \sim 5$, after which degrees of freedom that were integrated out perturbatively will receive non-perturbative contributions [11]. The breakdown will occur at smaller volumes for higher excited states.

For the Wilson action we have chosen $\beta = 3.0$ and $\beta = 2.4$; for the improved actions β was tuned to yield results in roughly the same physical volume. These parameters correspond to lattice spacings of approximately $a = 0.018$ and $a = 0.12$ fermi. For the smallest of these two, one expects tree-level improvement to be effective and we have therefore not tadpole corrected the actions in this case. Note that for these small volumes one finds from the analytic results that the lattice artefacts in the mass ratios are quite much bigger [14] than in larger volumes. Data was taken for both the LW and square Symanzik actions, and as a test on our programs also for the Wilson action for which we can compare with available high precision data [15].

At the larger volume we concentrated our attention

Table 1

Values of $z = mL$ at a lattice spacing of approximately 0.02 fermi for SU(2) on a $4^3 \times 128$ lattice. We have performed 16000 measurements (25 heat-bath sweeps apart) for Wilson and 20000 for both LW and square Symanzik actions (10 sweeps apart). The entries in the table correspond to the representations of the cubic group, the number of operators used in the variational analysis and the effective masses extracted from n/ℓ ratios of correlation functions, i.e. $-\log(\sum_t \langle \mathcal{O}(t+n)\mathcal{O}(t) \rangle / \sum_t \langle \mathcal{O}(t+\ell)\mathcal{O}(t) \rangle)$. Entries in boldface are taken as final estimates for Fig. 2. Errors have been analyzed using the jackknife method.

Rep.	# op.	1/0	2/1	3/2	4/3
Wilson action, $\beta = 3.0$					
A_1^+	5	2.105(9)	2.03(1)	2.02(2)	2.08(3)
E^+	3	1.743(5)	1.703(9)	1.71(1)	1.70(2)
T_2^+	3	3.315(9)	3.25(2)	3.21(4)	3.2(1)
e_1	3	0.277(3)	0.269(5)	0.269(6)	0.270(7)
e_2	3	0.588(5)	0.575(7)	0.576(9)	0.58(1)
e_3	3	0.978(5)	0.962(8)	0.97(1)	0.97(2)
LW Symanzik action, $\beta = 2.374$					
A_1^+	6	1.89(1)	2.01(2)	2.03(3)	2.09(5)
E^+	3	1.626(6)	1.77(1)	1.78(2)	1.80(3)
T_2^+	3	3.134(8)	3.43(2)	3.47(5)	3.4(1)
e_1	3	0.307(3)	0.334(5)	0.339(6)	0.342(8)
e_2	3	0.656(5)	0.718(8)	0.73(1)	0.74(2)
e_3	3	1.104(6)	1.21(1)	1.24(1)	1.26(2)
Square Symanzik action, $\beta = 2.2013$					
A_1^+	6	2.15(1)	2.31(2)	2.30(3)	2.29(5)
E^+	3	1.844(6)	2.00(1)	2.02(2)	2.04(3)
T_2^+	3	3.56(1)	3.87(3)	3.88(7)	3.8(3)
e_1	3	0.375(4)	0.405(6)	0.408(7)	0.412(9)
e_2	3	0.808(6)	0.877(9)	0.89(1)	0.90(2)
e_3	3	1.368(8)	1.49(1)	1.52(2)	1.54(9)

to the LW Symanzik action with tree-level and tadpole corrected one-loop values of the coefficients. We verified that there is no observable volume dependence of u_0 by comparing its value with the one on an $8^3 \times 64$ lattice (the difference was less than 0.3%, consistent with zero within statistical errors). Following the prescription of Refs. [4,7,9] we took for SU(2)

$$\begin{aligned}
 c_0 &= 5/3, & c_1 &= -(1 + 0.2227\alpha_s(u_0))/(12u_0^2), \\
 c_2 &= -0.02224 \times 5\alpha_s/(3u_0^2), & c_4 &= 0, \\
 \alpha_s(u_0) &\equiv -(4 \log u_0)/1.725969, & & (14)
 \end{aligned}$$

obtained from the one-loop coefficients determined by Lüscher, Weisz and Wohlert [3,12]. Substituting these coefficients in Eq. (1) and multiplying by $\beta/4 \equiv 1/g_0^2$ gives the action we used for our simulations. We do not

Table 2

The same as in Table 1 but for a lattice spacing of approximately 0.12 fermi. We have performed 40000 and 48000 measurements respectively for tree-level LW and tadpole corrected one-loop improved LW Symanzik actions, in both cases separated 2 heat-bath sweeps apart.

Rep.	# op.	1/0	2/1	3/2	4/3
LW Symanzik action, $\beta = 1.83$					
A_1^+	7	3.71(1)	3.74(2)	3.78(5)	3.9(2)
E^+	7	3.212(9)	3.29(2)	3.30(4)	3.3(1)
T_2^+	3	6.13(1)	6.31(6)	6.3(3)	6.2(1.0)
e_1	7	0.813(6)	0.84(1)	0.84(1)	0.84(2)
e_2	7	1.75(1)	1.80(2)	1.80(3)	1.80(3)
e_3	8	2.89(2)	3.05(3)	3.09(5)	3.2(1)
T_{11}^+	7	1.857(6)	1.92(1)	1.92(2)	1.91(3)
$T_2^+(111)$	7	2.67(1)	2.67(2)	2.66(3)	2.67(7)
Tadpole corrected LW Symanzik action, $\beta = 2.04$					
A_1^+	7	4.07(1)	4.06(3)	4.05(8)	3.8(2)
E^+	7	3.366(6)	3.57(2)	3.57(5)	3.6(1)
T_2^+	3	6.28(2)	6.76(7)	6.6(4)	6.5(1.0)
e_1	7	0.889(5)	0.94(1)	0.94(1)	0.94(2)
e_2	7	1.920(1)	2.02(2)	2.01(3)	1.98(4)
e_3	7	3.402(14)	3.41(3)	3.38(4)	3.21(7)
T_{11}^+	7	1.893(6)	2.06(1)	2.07(3)	2.07(4)
$T_2^+(111)$	7	2.79(1)	2.87(2)	2.86(4)	2.83(8)

absorb the tree-level value of c_0 in the definition of the coupling constant, as was done in Refs. [7,9]. When using the convention of Eq. (15) the standard two-loop relation between β and $a\Lambda$ needs no modification. But the Lambda parameter has to be corrected for the fact that the Lüscher-Weisz choice of coupling amounts to multiplying Eq. (1) by $(g_0^{-2} + 0.08112)$, so as to compensate for the one-loop correction to c_0 .

In the intermediate volumes we have used for our simulations, masses remain small compared to the spurious unphysical poles. This allows us to use the variational approach [16] to increase the overlap of the the states to be measured. We have been able to extract clean signals. On very coarse lattices where masses would no longer be small in lattice units, one loses the signal in the noise too early to extract it reliably, whereas also the variational method is no longer well founded. Recently these problems were tackled by using anisotropic lattices [9], well known from finite temperature studies [17]. Only implementing improvement for the spatial directions will in addition remove the problems with a non-hermitian transfer matrix.

The raw data are listed in Tables 1 and 2, based on

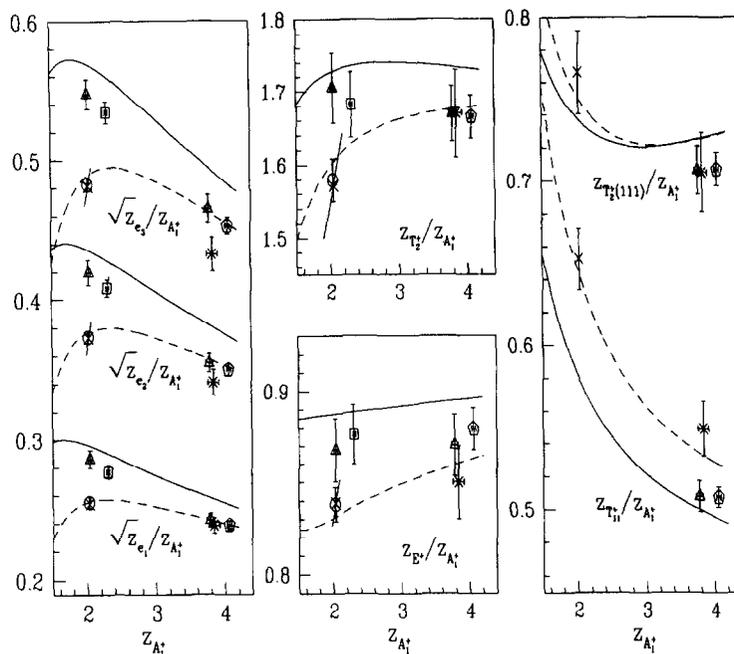


Fig. 2. SU(2) Monte Carlo data (see tables) on a $4^3 \times 128$ lattice for the Wilson action (circles for our data and crosses for data by Michael [15], with tilted error bars when data overlap), the LW Symanzik improved action (triangles), the square Symanzik action (squares) and the tadpole corrected one-loop LW Symanzik action (pentagons) at lattice spacings of approximately 0.02 and 0.12 fermi. A comparison is made with analytic results for the continuum (solid lines) and Wilson action on a lattice of size $4^3 \times \infty$ (dashed lines).

performing the variational analysis on the second time slice. We have verified that the result is stable against performing the variational analysis on the first time slice. We used 3 to 8 operators, as defined in Ref. [15], for the variational analysis. They were computed in terms of Teper-fuzzed links [18]. Only for the determination of the scalar glueball mass at $a \sim 0.02$ fermi the variational analysis was important, in most other cases a single *but Teper-fuzzed* operator was sufficient to obtain accurate results.

Another issue is that for $a \sim 0.02$ fermi the small value of the coupling gives rise to large autocorrelations that can affect the energies of electric flux. In most cases we found it useful to correct for this by eliminating data for which the average of the spatial Polyakov loops over the 128 time slices (and a few heat bath updates) was bigger in absolute value than one half. Our results for the Wilson case at $\beta = 3$ agree to high accuracy with those reported by Michael [15].

Because of the availability of analytic results, it is not necessary to exactly tune the different actions to the same physical volume. Nevertheless in particular for the data at $a = 0.12$ fermi we made an effort

to tune parameters appropriately, as we can make a stronger point when directly comparing lattice data at the same physical volume. The value of u_0 is determined self consistently [7,9], adjusting with the help of the Ferrenberg-Swendsen trick [19] the input value of u_0 to agree with its measured value. This only requires little Monte Carlo time. The results of Tables 1 and 2 are presented in Fig. 2 to compare with the analytic results for the continuum (solid curves) and for the Wilson action on a lattice of size $4^3 \times \infty$. We have used approximately 160 hours of CPU time on a Cray C98 to generate and analyze the data presented in this paper. Computational overheads for improved actions amount to a factor 3 for the LW and 4 for the square Symanzik action over the standard Wilson action.

4. Discussion

As was to be expected, at lattice spacings of approximately 0.02 fermi ($z_{A_1^+} \sim 2$), tree-level improvement is seen to bring the lattice results quite close to those of the continuum, both for the LW and square

Symanzik actions. In both cases the improvement is considerable.

Also at lattice spacings around 0.12 fermi and volumes of approximately 0.48 fermi ($z_{A_1^+} \sim 4$), the agreement of the Wilson action lattice data with the corresponding analytic results is in general very good for the lowest lying states. The difference in the analytic result between the continuum and Wilson lattice action gives an indication how far the improved data is removed from the continuum result. Significant improvement is observed in some of the cases, in particular for $z_{T_{11}^+}/z_{A_1^+}$, approaching the continuum analytic result.

The most salient feature of our data is that tadpole correction has no significant effect on the tree-level improved data for the ratios. Perhaps for the cases where tree-level improvement is already significant this is what one would want, but our results show some instances where tree-level improvement has no effect *and* the tadpole correction is of no help either.

In particular we note that the ratio $\sqrt{z_{e_1}}/z_{A_1^+}$, measured to an accuracy of better than 1.5%, deviates from its continuum value by 5–6%. For this quantity tree-level improvement *as well as tadpole corrected* one-loop improvement are unable to show deviations from the Wilson result. This result puts some doubt on the usefulness of the tadpole correction for careful extrapolations of mass ratios to the continuum limit.

One might object that the lattice spacing we have used to implement the tadpole correction, $a = 0.12$ fermi, is not really large enough. We have certainly not probed lattice spacings as large as $a = 0.4$ fermi, that have been advertised [7]. Nevertheless for $a = 0.12$ fermi, $u_0^4 = 0.6819(1)$ and significantly deviates from 1. The correction to c_1 at these parameters is 27% with respect to its tree-level value (without tadpole correction it would have been 17%).

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